



Evaluating Single and Multi-Server Exponential Queuing Models: A Case Study of Access Bank PLC in Anyigba, Kogi State, Nigeria

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KEYWORDS

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ABSTRACT

Queuing takes place when the number of individuals waiting in line surpasses the system's maximum capacity, this happens when the line extends beyond the available servers. The banking sector in Nigeria is facing challenges related to prolonged queues, adversely impacting the nation's economic growth. This issue results in customer dissatisfaction thereby hindering productivity and complicating patronage. Also, there is an economic loss to each person while remaining in the queue, which makes it essential to minimize, if not eliminate, the challenges of long queues in Nigerians banking system. This article assesses both single and multi-server exponential queuing models. Primary data was collected through direct observation and personal interviews, recording inter-arrival times and service durations from customer service unit of Access Bank Plc, Anyigba Kogi State. Performance indicators for both single and multi-server queuing models, such as utilization factor, average queue length, average system length, average queue waiting time, and average system waiting time, were computed and analysed. The result revealed that the (M/M/S) : (FCFS/∞/∞) model outperforms the (M/M/1) : (FCFS/∞/∞) model by minimizing customer waiting time from approximately 2.0 minutes to 0.03 minutes. The findings emphasized the efficiency of employing multiple servers, this shows that introducing more servers reduces the workload per server, potentially attracting more customers. Furthermore, a comprehensive analysis of cost implications and utilization factors served as a target for achieving a balance between minimizing cost and ensuring an optimal server level at the customer service of Access Bank Plc. The results indicated that for an optimal balance between service level and total cost, adopting the (M/M/4) : (FCFS/∞/∞) model is recommended, as it results in a lower cost of ₦9,104.99 compared to ₦10,793.43.

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1. INTRODUCTION

Queuing is the process of lining up customers to place orders, with the term "queue" originating from the French word *queue*, meaning "to line up." Queuing can involve human customers or physical entities waiting for service in places such as petrol stations, banks, mechanic workshops, airports, car parks, and goods for shipment. In Nigeria, queuing is a common experience, especially in banks and fuelling stations [1]. Queuing occurs when the number of individual items waiting in line exceeds the maximum capacity of the system. This happens when the line grows longer than the available servers. The line is made up of individuals who are waiting to complete a task, process or receive a service, and the number of people

waiting goes beyond the limit set by the system's capacity. The main causes of long queues include inadequate service systems and low service quality which increase the waiting time. In service-related industries, reducing waiting time and providing prompt service are critical factors in enhancing customer satisfaction which leads to improved service quality [1].

Queuing theory is a mathematical study of waiting lines and their associated problems. Service operates on a First-In-First-Out (FIFO) basis, with customer being served one at a time. The queuing theory is a quantitative analysis technique used to predict the characteristics of a waiting line. It enables the mathematical analysis of queuing behavior, including customer arrival time and the amount of time a customer waits

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in the system in a real-world queuing situation [2]. By applying the queuing theory, companies can estimate and improve their service capacity, which helps them provide better service quality and reduce waiting time [3].

Waiting time refers to the amount of time that a customer spends in a queue before receiving service [2]. Waiting for service is a common occurrence in various customer-centric settings, including gas stations, banks, hospitals, restaurants, and departmental stores [4]. When customers visit a service facility, they may need to wait for their turn, which can be frustrating if the waiting time is long. Such situations can lead to customer dissatisfaction, as they are unable to receive the level of service they desire [5]. Moreover, some customers may faint or even die while waiting in the queue. Service delays occur when the need for a service surpasses the existing capacity, resulting in the formation of a queue [5].

Service time in a queuing model refers to the amount of time it takes to serve a customer or process a task at a service point, such as a checkout counter or a customer service desk. It is a critical factor in analyzing and modeling queues or waiting lines. Service time can vary based on factors like the complexity of the task, the efficiency of the service, and the nature of the customers or tasks in the queue. In queuing theory, understanding service time distribution helps in predicting waiting times, queue lengths, and overall system performance.

1.1 Problem Statement

The banking sector in Nigeria faces challenges related to prolonged queues due to inadequate queue management. This issue has detrimental effects on the country's economic growth and development. The extended waiting times lead to customer dissatisfaction and often prompt them to leave the system prematurely, hindering productivity and complicating customer engagement. In the increasingly competitive banking environment, customers expect high-quality, cost-effective, and prompt service delivery. Waiting is perceived as inconvenient, and the associated time becomes a tangible cost to customers. Moreover, the extended waiting period incurs an economic loss for individuals in the queue, necessitating the minimization, if not the elimination, of challenges posed by long queues in the country's banking system. To ensure the efficient operation of banking services and enhance customer satisfaction, it is imperative to adopt strategies that reduce waiting times. To address these concerns, this article concentrates on customer services, utilizing single and multi-server exponential queueing models to analyze queues at Access Bank Plc in Anyigba, Kogi State.

1.2 Objectives

The objectives of this study is to assess single and multi-server exponential queueing models for Access Bank Plc, Anyigba, Kogi State; to determine the arrival rate and service rate of customers; to examine the average waiting time of the customers in queue; to estimate the waiting cost of customers and the service cost of the bank facilities and to identify the best queueing model for the bank.

1.3 Literature Review

Prior to the investigations carried out by earlier scholars in the field, [6] explored queueing theory and customer satisfaction at the First Bank Plc Gwagwalada, Abuja branch. [3] proposed that completely eliminating queues in a system might moderately diminish the perceived value of attraction. [4]

scrutinized how queueing theory could be employed to analyze operations at three commercial banks, particularly focusing on the teller counter and Automated Teller Machines (ATM) areas. [7] investigated into the performance of ATMs in Nigerian banking institutions, with a specific emphasis on the operations of First Bank of Nigeria Plc in Ibadan. [8] applied queueing theory to scrutinize vehicular traffic on the Nakuru Total Road Stretch, while [9] incorporated cost functions into queueing theory to evaluate the ideal service level of the queueing system at Guaranty Trust Bank in Jos, Nigeria. [10] conducted an analysis at a public health clinic in southern Malaysia, using queueing theory to scrutinize and simulate patient flow at the outpatient department. [11] conducted a study on the utilization of queueing models to minimize waiting times at fuel stations. [12] undertook a comprehensive investigation exploring the influence of applying queueing theory models on productivity performance within the banking sector. [13] conducted research at First Bank Plc, Kaura Namoda Branch in Zamfara State, concentrating on queue theory and ATM service optimization. [14] focused on applying queueing theory to the banking system in the Democratic Republic of the Congo, specifically examining BCDC (Banque Commerciale du Congo) Mbuji-Mayi. [15] suggested a study to identify the most effective queueing strategy for airport taxis and passengers in a scenario with a high number of taxis and passengers. [16] applied a mathematical model to examine waiting times at two specific banks in the Sekondi-Takoradi Metropolis. [17] investigated the implementation of single and multi-server exponential queueing models in several selected hospitals in the north-western region of Nigeria.

In addition, [18] conducted a study to investigate the utilization of queue theory in the Nigerian banking system. The focus was on GTBank and the Ecobank Idumota branch in Lagos, Lagos State. The study employed a multi-server model to analyze the characteristics of the queueing system in these banks. [19] conducted a research comparing the queue management systems of First Bank Plc. and Guaranty Trust Bank Plc, located in Isokun Ilesa, Nigeria. [20] investigated various aspects of queueing theory to examine the waiting time of patients in hospitals. They examined parameters such as the rate at which patients arrive, the rate at which services are provided, the utilization factor, the average number of patients in the system, the average number of patients in the queue, the average time spent by patients in the system, and the average time spent by patients in the queue. [12] conducted a research study aiming to address the persistent issue of queueing in the banking sector. They conducted a comparative analysis to determine the impact of various queueing approaches employed by certain banks, including Artificial Neural Network (ANN), Business Process Reengineering (BPR), M/M/1, M/G/1, and the Erlang B and C formulas. [21] conducted a study on Bank Queueing Optimization using Markov Process. The researchers focused on analyzing the arrival of customers and the service provided by cashiers to create a birth-death process and its transfer rate matrix. They introduced the concepts of service intensity and customer satisfaction index and found that it is possible to calculate and predict the optimal number of open cashiers for different time periods. [22] investigated a study with the aim of assessing queue management practices and patient satisfaction at specific hospitals in Northwestern Nigeria. [23] conducted a study using the queueing management approach to investigate the unpredictable arrival pattern of customers at a service system. To prevent customers from

abandoning the queue, they employed an Artificial Neural Network-based waiting time predictor. This method enables the system to anticipate and generate the expected waiting time for each customer. Consequently, customers have the option to engage in other activities instead of waiting in line until it's their turn.

2. METHODOLOGY

Primary data is used; observations were made at the customer service unit of Access Bank Plc, Anyigba, Kogi State. The data collected were the customer's time of arrival, time spent waiting in the queue, time spent receiving services, and then the time of departure from the bank. These help in the estimation of the average number of customers that entered the bank per hour and the average number of customers that were attended to per hour. The data collected covers a period of two weeks, in which five days of the week from Monday to Friday were considered the working days of the week from 8.00 a.m. to 4.00 p.m. A customer is considered to have arrived when he or she joins the queue at the customer service unit. The waiting time in the queue ended immediately after the customer gained access to the bank service. Also, the service time was recorded from the time the customer gained access to the bank service to the time of exit. The queue discipline to observe is First Come, First Serve (FCFS). The data analysis is carried out using R software. Time records obtained in the process of data collection were entered into Microsoft Excel for conversion of the recorded time into interval time. The time interval is computed to get the mean arrival rate α and the mean service rate μ , which will then be entered into R software to create scenarios different from the real-life observations obtained from the process of data collection. The software, as stated earlier, is used to evaluate the performance measures of the queuing system.

2.1 The Single Queue Single-Server (M/M/1) Model

This model has Poisson arrival, Poisson service, exponential inter-arrival /service time. Single channel, Infinite system capacity and First-Come- First -Serve queue discipline. The letter "M" here is used to honour Russian mathematician Andreyevich Markov, who has published extensively on queuing theory [25]. It is used to represent both the inter-arrival and service time distributions in Kendall's notation.

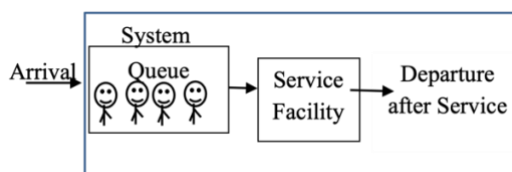


Fig. 1. A typical single-server queuing process

2.1 Characteristics of M/M/1 Queue Model

The M/M/1 queuing model means that the arrival and service time are exponentially distributed (Poisson process). In most cases, it is useful to determine the various waiting times and queue sizes of the queuing system in order to determine if the queue is in a good state or suggest better models for the queue. Little's formula is an equation that shows that the average number of customers in a queuing system is equal to their average arrival rate multiplied by the average amount of time spent in the system [24].

The average server utilization, ρ is given by:

$$\rho = \frac{\alpha}{\mu} \quad (1)$$

Equation (1) is the proportion of time that a server actually spends with a customer where, α is the average number of customers arriving per unit of time and μ is the average number of customers completing the service per unit time.

$$P_0 = \frac{\mu - \alpha}{\mu} \quad (2)$$

Equation (2) is the probability of having no customer in the system.

$$P_k = \left(\frac{\alpha}{\mu}\right)^k P_0 \quad (3)$$

Equation (3) is the probability of having k customers in the system. Where P_0 is as defined in equation (2), k is the number of customers.

$$L_s = \frac{\alpha}{\mu - \alpha} \quad (4)$$

Equation (4) is the average number of customers in the system.

$$L_q = \frac{\alpha^2}{\mu(\mu - \alpha)} \quad (5)$$

Equation (5) is the average number of customers in the queue

$$W_q = \frac{\alpha}{\mu(\mu - \alpha)} \quad (6)$$

Equation (6) is the average waiting time in the queue.

$$W_s = \frac{1}{\mu - \alpha} \quad (7)$$

Equation (7) is the average time spent in the system, including the waiting time.

2.2 The Single Queue Multi-Server (M/M/S) :(FCFS/ ∞ / ∞) Model

The (M/M/S) :(FCFS/ ∞ / ∞) model describes a scenario where there are multiple service facilities operating simultaneously, all providing the same service. In this format, several customers in the waiting line can receive service from any of these stations. It is assumed that customer arrivals and service times are distributed according to the Poisson and exponential distributions, respectively. Multiple channels, Infinite system capacity and First-Come- First -Serve queue discipline

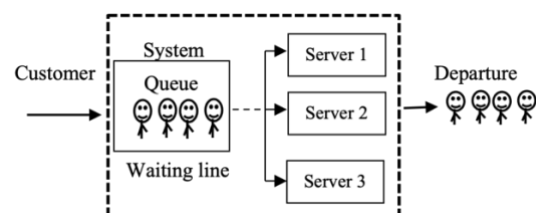


Fig. 2. A typical multi-server queuing process

2.3 Characteristics of (M/M/S) :(FCFS/∞/∞) Model

- (i) If, $k \geq S$, every one of the servers are occupied, and the maximum number of customer waiting in the queue will be $(k - S)$, then $\mu_k = S\mu$. Additionally, $\alpha_k = \alpha$ for $k = 0, 1, 2, 3, \dots$
- (ii) If $k < S$, every one of the customer might be served concurrently and there will be no queue. $S - k$ number of servers is likely to be idle and then $\mu_k = k\mu$ for $k = 0, 1, 2, 3, \dots k$

$$\rho = \left(\frac{\alpha}{S\mu}\right) \quad (8)$$

There are S servers in parallel, i.e. an M/M/S system but the expected capacity per time is the $S\mu$ in equation (8)

$$P_k = \begin{cases} \frac{\rho^k P_0}{k!}; & k < S \\ \frac{\rho^k P_0}{S^{k-S} S!}; & k \geq S \end{cases} \quad (9)$$

Where P_k is the probability of having k unit (customer) in the system.

$$P_0 = \left[\sum_{k=0}^{S-1} \frac{1}{k!} \left(\frac{\alpha}{\mu}\right)^k + \frac{1}{S!} \left(\frac{\alpha}{\mu}\right)^S \frac{S\mu}{S\mu - \alpha} \right]^{-1} \quad (10)$$

Equation (10) is the probability of having no customer in the system.

Expected number of customers in queue (L_q)

$$L_q = \left[\frac{1}{(S-1)!} \left(\frac{\alpha}{\mu}\right)^c \frac{\alpha\mu}{(S\mu - \alpha)^2} \right] P_0 \quad (11)$$

Expected number of customers in the system (L_S)

$$L_S = L_q + \rho \quad (12)$$

The average time a customer waits for service (W_q)

$$W_q = \frac{L_q}{\alpha} \quad (13)$$

The average time a customer spent is in the system, (W_S)

$$W_S = \frac{L_S}{\alpha} \quad (14)$$

2.4 Introduction of Cost into the Queuing model

In order to determine the optimal service level and cost implications of a given queuing system [26] stated that we have to contend with two cost namely:

- Customers' Waiting Costs (Cost of delay in offering service)
- Service cost (Cost of offering service)

Let the Expected Waiting Cost E(WC) per unit time be defined as:

$$E(WC) = \alpha W_S C_w \quad (15)$$

Where W_S is as defined in equation (7), C_w is the opportunity cost for customer waiting in the system.

Also, Let the Expected Service Cost E(SC) be defined as:

$$E(SC) = C_0 + S C_S \quad (16)$$

Where C_0 is the fixed cost per unit time, S is the number of servers, C_S is the cost per server.

In order to obtain the total cost of the system, we add the waiting and service cost in equations (15) and (16). Hence, the E(TC) is defined as:

$$E(TC) = \alpha W_S C_w + C_0 + S C_S \quad (17)$$

Total cost model in equation (17) attempts to create a balance between the two conflicting costs namely: the waiting and service cost, because an increase in one cost automatically causes a decrease in the other [19].

3. RESULTS AND DISCUSSIONS

In this section, we delve into the analysis and discussion. The data was obtained from the customer service unit of Access Bank Plc, Anyigba, Kogi State, Nigeria, over the period of two weeks.

3.1 Service Utilization Factor (ρ)

This performance metric signifies the proportion of time during which an equipment or system is actively operating in relation to the total time it could potentially be in use $\rho = 0.9517$, this indicates that the queuing system is operational for about 95.17% of the time and remains inactive merely for 4.83% of the time.

Table 1. Presents a condensed overview of data collection spanning two-week period.

Day	Week 1		Week 2	
	α_1	μ_1	α_2	μ_2
Monday	10.097	10.71	8.899	9.697
Tuesday	9.144	10.81	9.21	9.647
Wednesday	9.737	8.849	9.019	9.402
Thursday	9.599	10.12	9.441	10.242
Friday	9.267	8.905	9.322	10.109
Total	47.844	49.394	45.8913	49.097
Average	9.5688	9.8788	9.17826	9.8194

3.2 Performance Measure of M/M/S:FCFS/ ∞ / ∞

Table 2 presents a comparison of different queuing systems, ranging from M/M/1 to M/M/6, and assesses their performance using various metrics such as utilization factor (ρ), Probability of having no customer in the system (P_0), average number of customer in the queue (Lq), average number of customer in the system (Ls), average waiting time in the queue (Wq) and average time spent in the system (Ws) for both single-server and multi-server queuing models as presented in the methodology. The result revealed that the average customer waiting times in the system (Ws) per minute decrease as the number of servers increases: 0.2134, 0.1471, 0.0832, 0.0585, 0.0453, and 0.0369 for 1, 2, 3, 4, 5, and 6 servers, respectively.

This can be generalized and applied to other banks or sectors facing similar challenges to optimize customer service and operational efficiency. The results demonstrate that as the number of servers increases, the average customer waiting time per minute significantly decreases from 0.2134 minutes with one server to 0.0369 minutes with six servers indicating that additional servers lead to fewer customers waiting in line. Concurrently, there is a notable decrease in the service utilization rate, dropping from 95.17% with one server to 15.83% with six servers, which suggests that each server becomes less busy as more servers are added. This reduction in both waiting times and server utilization implies that increasing the number of servers can enhance customer satisfaction by minimizing wait times and improving the overall customer experience, potentially leading to higher customer patronage. Consequently, other banks and sectors with high customer traffic can adopt similar strategies, adjusting their number of service points or servers to balance customer wait times and server utilization, thereby improving service quality and operational efficiency.

3.3 Cost Analysis

To determine the marginal cost, hourly charges were employed to maintain consistency in the calculations.

- i. For server cost (Cs) per hour, the monthly earned allowances for two staff are at a flat rate of N200,000. The server cost (Cs) per hour = $200,000/30 \text{ days}/8 \text{ hrs}$ = N833.33.

- ii. The server attends to the customers between 08:00 a.m. and 04:00 p.m. (total of 8 hours in a day).
- iii. The waiting cost per customer per hour is N2,479.7.
- iv. Bank management claimed to spend on an hourly basis N5,000 on the maintenance and servicing of queuing facilities, e.g., desktops, cooling systems, sanitation, and other equipment.

Table 3 provides a breakdown of the cost analysis for both single-server and multi-server queue models. The computation involved considering server (staff) salary structure, waiting cost, and service cost to identify the model with the lowest overall cost.

In the single-server model (M/M/1):(FCFS/ ∞ / ∞), the total cost is ₦ 10,793.43, resulting in 95.17% server utilization, indicating a high degree of overutilization at 95%.

For multi-server models (M/M/s):(FCFS/ ∞ / ∞), the cost for (M/M/2) is ₦ 10,085.74 at a 47.49% utilization level. Subsequent models, (M/M/3), (M/M/4), (M/M/5), and (M/M/6), incur costs of ₦ 9,433.83, ₦ 9,104.99, ₦ 10,219.57, and ₦ 10,857.65, respectively, with decreasing utilization levels.

The results highlight that, at the time of this study, the single-server model (M/M/1) (FCFS/ ∞ / ∞) has the highest total cost at ₦ 10,793.43. As the number of servers increases, the system's cost decreases, reaching ₦ 9,104.99 for the (M/M/4):(FCFS/ ∞ / ∞) model, which achieved a 23.74% utilization level. Striking a balance between waiting time and service level, this model represents the optimal service level with the minimum total cost. Figure 3 above shows the overall cost of the system concerning both single and multiple server models. The chart indicates that the most favorable service level is attained when employing the multi-server model with four servers.

4. CONCLUSION

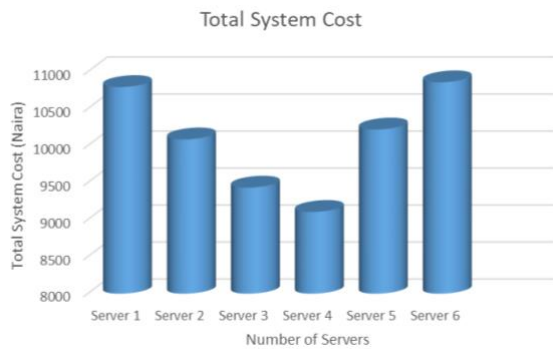
This article evaluates the importance of utilizing single and multi-server exponential queuing models for Access Bank Plc in Anyigba, Kogi State, Nigeria.

Table 2. Performance indicators for single and multi-server queuing models of customers in Access Bank Plc, Anyigba

Performance measures	M/M/1	M/M/2	M/M/3	M/M/4	M/M/5	M/M/6
α	9.3734	9.3734	9.3734	9.3734	9.3734	9.3734
μ	9.8491	9.8491	9.8491	9.8491	9.8491	9.8491
ρ	95.17%	47.49%	31.66%	23.74%	18.99%	15.83%
P_0	0.0502	0.5251	0.6834	0.7626	0.8100	0.8417
Lq	18.9066	0.9043	0.4632	0.3114	0.2345	0.1881
Ls	19.7044	1.3792	0.7798	0.5488	0.4244	0.3464
Wq	2.0006	0.0965	0.0494	0.0332	0.0250	0.0201
Ws	0.2134	0.1471	0.0832	0.0585	0.0453	0.0369

Table 3. The Computation of Cost Analysis

S	α	W_s	αW_s	C_0	C_s	C_w	CC_s	$E(W_c)$	$E(S_c)$	$E(TC)$
1	9.3734	0.2134	2.000284	5000	833.33	2479.7	833.33	4960.103	5833.33	10793.43314
2	9.3734	0.1471	1.378827	5000	833.33	2479.7	1666.66	3419.078	6666.66	10085.73766
3	9.3734	0.0832	0.779867	5000	833.33	2479.7	2499.99	1933.836	7499.99	9433.825902
4	9.3734	0.0332	0.311197	5000	833.33	2479.7	3333.32	771.6749	8333.32	9104.994903
5	9.3734	0.0453	0.424615	5000	833.33	2479.7	4166.65	1052.918	9166.65	10219.56787
6	9.3734	0.0369	0.345878	5000	833.33	2479.7	4999.98	857.6748	9999.98	10857.65482

**Fig. 3.** The overall cost of the system

The study reveals that both inter-arrival and service times adhere to the $(M/M/1) : (FCFS/\infty/\infty)$ general distribution model, employing the First-Come, First-Serve (FCFS) queuing discipline. The results indicate that the $(M/M/s) : (FCFS/\infty/\infty)$ multi-server model outperforms the $(M/M/1) : (FCFS/\infty/\infty)$ single-server model. This improvement is evident in average queue and system sizes, as well as average waiting times, particularly when there is an increase in the number of servers. The findings underscore the efficacy of employing multiple servers to enhance efficiency. Additionally, a thorough examination of cost implications and utilization factors serves as a benchmark for achieving a balance between cost minimization and ensuring an optimal service level at Access Bank Plc's customer service.

The study highlights the importance of considering both cost-effectiveness and service quality to enhance customer service delivery. Based on the research outcomes, the following recommendations are proposed: It is advisable to implement the $(M/M/4) : (FCFS/\infty/\infty)$ multi-server model, as it effectively reduces customer waiting time, improves service delivery, and significantly reduces operational costs. Promoting online banking is also recommended to alleviate pressure on customer care services.

Implementing the $(M/M/4) : (FCFS/\infty/\infty)$ multi-server model, while effective in reducing customer waiting times, improving service delivery, and lowering operational costs, may face several challenges and limitations. One potential challenge is the initial investment required to establish and maintain multiple servers, including the costs associated with infrastructure, technology, and staffing. Additionally, managing a multi-server system can be complex, requiring robust scheduling and coordination to ensure that servers are efficiently utilized without causing downtime or overstaffing during off-peak hours. There is also the risk of underutilization of resources if customer demand fluctuates

significantly, which could lead to increased costs without proportional benefits. Furthermore, the model assumes an infinite queue capacity and steady arrival rates, which may not always align with real-world scenarios where customer flow can be unpredictable and vary widely. Lastly, customer satisfaction depends not only on reduced wait times but also on the quality of service provided, which requires continuous training and management oversight. Therefore, while the $(M/M/4)$ model presents a promising strategy, its implementation must be carefully planned and monitored to address these potential challenges and ensure sustainable improvements in service delivery and cost efficiency.

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