MJSAT

Malaysian Journal of Science and Advanced Technology

journal homepage: https://mjsat.com.my/

MCDM based on Trapezoidal Neutrosophic Numbers for Biogas Recovery Substrates Selection Problems

Norzanah Abd Rahman*[1], Rafidah Selaman[2], Zamali Tarmudi[3], and Nor Hashimah Sulaiman[4]

[1] *Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Sabah Branch, Kota Kinabalu Campus, Sabah, Malaysia.*

[2] *Faculty of Applied Sciences, Universiti Teknologi MARA Sabah Branch, Kota Kinabalu Campus, Sabah, Malaysia.*

[3] *Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Johor Branch, Segamat Campus, Johor, Malaysia.*

[4] *Centre of Foundation Studies, Universiti Teknologi MARA, Selangor Branch, Dengkil Campus, Selangor, Malaysia.*

KEYWORDS

Fuzzy Set;
Neutrosophic Set;
Trapezoidal Neutrosophic Numbers;
Multi-Criteria Decision Making method;
Biogas Recovery

ARTICLE HISTORY

Received 7 June 2025
Received in revised form 17 November 2025
Accepted 10 February 2026
Available online 27 March 2026

ABSTRACT

In the context of decision-making under uncertainty, fuzzy set theory has been extensively applied to model imprecise and ambiguous information. However, the absence of a falsity membership function limits its ability to fully capture the complexities of uncertain data. Neutrosophic Set (NS) theory addresses this limitation by introducing the truth, indeterminacy, and falsity functions, enabling a more comprehensive representation of uncertain, indeterminate, and inconsistent information. Despite the theoretical advancements of NS theory, its application in scientific decision-making remains limited. To bridge this gap, this paper proposes a Multi-Criteria Decision Making (MCDM) method based on Trapezoidal Neutrosophic Numbers to effectively address the uncertainty associated with substrate selection for biogas recovery. The result shows that among five biogas substrates, Palm Oil Mill Effluent Sludge is the most suitable substrates for biogas with a score of 1.4316. This is followed by Palm Oil Mill Effluent with a score of 1.0868, Carbohydrate-Rich Food Waste with 0.9081, Protein-Rich Food Waste with 0.7868, and Fiber-Rich Food Waste with 0.7410. In addition, the sensitivity analysis shows that the ranking remains stable under input variations, while comparative analysis confirms the robustness and consistency of the proposed methodology against existing approaches. These findings demonstrate that the MCDM based on Trapezoidal Neutrosophic Numbers offers a structured and reliable framework for handling uncertainty, offering practical value to decision-makers in biogas recovery and similar domains.

1. INTRODUCTION

Decision-making under uncertainty remains a significant challenge across scientific disciplines, particularly in selecting biogas recovery substrates. However, current uncertainty modeling tools often fall short in addressing the complex variables inherent in biogas processes, as they fail to adequately represent the unique characteristics of organic waste substrates.

To address this gap, traditional fuzzy set theory, as introduced by Reference [1], provides a mathematical framework for modeling imprecise and uncertain information using a membership function. Building on this foundation, subsequent developments such as the Intuitionistic Fuzzy Set (IFS) theory proposed by Reference [2] expand this framework by incorporating both membership and non-membership functions. Further advancing this approach, Neutrosophic Set (NS) theory, introduced by Reference [3], adds an indeterminacy membership function in addition to truth and falsity functions. This enhancement enables a more precise representation of incomplete, inconsistent, and ambiguous

*Corresponding author:
E-mail address: Norzanah Abd Rahman < norza9522@uitm.edu.my >.
https://doi.org/10.56532/mjsat.v6i1.538
2785-8901/

information, thereby enhancing the modeling of complex uncertainty.

Since then, NS theory has been further generalized into various forms, including Interval-Valued Neutrosophic Sets [4], Single-Valued Neutrosophic Sets [5], Generalized Neutrosophic Soft Sets [6], Rough Neutrosophic Sets [7], Simplified Neutrosophic Sets [8], Bipolar Neutrosophic Set [9], and Multi-Valued Neutrosophic Sets [10], [11], and Trapezoidal Neutrosophic Sets [12].

To effectively apply the theory of NS in decision-making scenarios, researchers have developed the Single-Valued Trapezoidal Neutrosophic Numbers (SVTNNs), as discussed in [13]. Prior research explores SVTNN operation and properties, including the ranking method [14], [15], [16], aggregation operators [17], [18], distance measures [19], and defuzzification techniques [20]. The researchers integrated the NS concept into the Multi-Criteria Decision Making (MCDM) problems, such as multi-robot systems [21] and the transportation problem [22].

While Single-Valued Trapezoidal Neutrosophic Numbers (SVTNNs) have been applied to various Multi-Criteria Decision Making (MCDM) problems, their fixed single-valued structure limits their ability to represent high granularity uncertainty. In contrast, Trapezoidal Neutrosophic Numbers (TrNNs), as introduced by Reference [12] and [23], enable an interval-based representation for each membership function. This approach offers greater flexibility and precision in modeling data from variable or imprecise decision-making processes. Applications of TrNNs in MCDM methods include the Analytic Hierarchy Process (AHP) and DELPHI [24], the Technique for Order of Preference by Similarity to Ideal Solution [25], and the VIekriterijumsko KOmpromisno Rangiranje (VIKOR) [26].

Although fuzzy set theory has been applied in various decision-making contexts, its use in scientific domains remains relatively limited. In biogas recovery research, a range of fuzzy-based and hybrid intelligent approaches (e.g., fuzzy logic and expert system, fuzzy DEMATEL, fuzzy modeling ANFIS, genetic algorithm-optimized NAFIS, and hybrid fuzzy-decision tree model) have been applied for diverse purposes, including operational efficiency assessment [27], process parameter optimization [28], biochemical methane potential parameter classification [29], byproduct (H2S) control [30], and biogas yield prediction [31].

However, the application of NS theory in biogas recovery studies remains unexplored. Yet, the problem of substrate selection for anaerobic digestion (AD) is inherently uncertain, as it depends on key characteristics such as Total Solid (TS) and Volatile Solid (VS), which vary across different organic wastes. Identifying the most suitable substrate is crucial, as it directly affects biogas yields, greenhouse gas mitigation, and the efficiency of renewable energy production [32]. A substrate is deemed suitable for biogas recovery if it meets three criteria: total solids (TS) content exceeding 30% to ensure an adequate organic loading, volatile solids (VS) content exceeding 70% reflecting the proportion of degradable matter for microbial conversion, and a potential of hydrogen (pH) level within the range of 6.8 to 7.2, which supports the methanogenic activity and prevents inhibition. These thresholds are consistent with findings in the literature. Several studies have emphasized the strong correlation between VS fraction [33], while the importance of maintaining the pH for stable methanogenesis has also been highlighted [34]. Furthermore, the TS content above 30% is typically required in the AD process [35].

To address this research gap, this study proposes an MCDM method based on TrNNs to evaluate and rank substrates for biogas recovery. The assessment considers five alternatives: Carbohydrate-Rich Food Waste (CRFW), Fiber-Rich Food Waste (FRFW), Protein-Rich Food Waste (PRFW), Palm Oil Mill Effluent Sludge (POMES), and Palm Oil Mill Effluent (POME), evaluated using key biochemical criteria under uncertainty. The framework integrates TrNNs with both arithmetic and geometric aggregation operators and utilizes a score function to rank the alternatives. To ensure objectivity and transparency in the evaluation process, the criterion weights were determined through an expert survey, in which a domain expert provided insights into the importance of each criterion. This approach not only reinforces the methodological transparency but also aligns with the goal of achieving a robust decision-making framework.

The biogas recovery substrate selection process is demonstrated and subsequently validated using sensitivity analysis. To further evaluate the consistency and robustness of the proposed methodology relative to existing approaches, a comparative analysis is conducted.

The main contributions of this study are as follows:

1. A novel application of TrNNs to biogas substrate selection, addressing uncertainty, indeterminacy, and inconsistency in decision-making.
2. A structured MCDM framework that integrates aggregation (arithmetic and geometric) and ranking method (score function) methods to provide reliable evaluations.
3. Validation of the methodology through sensitivity and comparative analyses, which demonstrates its stability, consistency, and practical relevance for renewable energy decision-making problems.

The remainder of this paper is arranged as follows: Section 2 introduces the preliminaries of fuzzy sets, NS theory, TrNNs, and defuzzification and aggregation methods. Section 3 presents the proposed methodology. Section 4 discusses the application results, including sensitivity and comparative analyses. Section 5 concludes with the main findings and implications of the study.

2. PRELIMINARIES

This section presents the preliminaries of fuzzy sets, defuzzification, and aggregation methods. Let X be the universe discourse,

2.1 Fuzzy Set

Definition 1 Fuzzy Set [1]: A fuzzy set, denoted by, F is defined as: $F=\{\left(x, \mu_F(x)\right) | x \in X\}$ where μ_F: X→ [0, 1] is the membership function of F. The membership value $\mu_F(x)$ describes the degree of belonging to x ∈ X in F.

Definition 2 Trapezoidal Fuzzy Number [36]: A trapezoidal fuzzy number, denoted by, $\tilde{F}$ is defined by its membership function as:

$$\mu_{\tilde{F}}(x)=\begin{cases}\frac{x-\alpha}{\beta-\alpha}\omega & \alpha\le x\le\beta\\ \omega & \beta\le x\le\gamma\\ \frac{\delta-x}{\delta-\gamma}\omega & \gamma\le x\le\delta\\ 0 & \text{otherwise}\end{cases}$$

2.2 Neutrosophic Set

Definition 3 Neutrosophic Set [3]: A Neutrosophic Set, denoted by, N is defined as: $N=\{\langle x,(T(x), I(x), F(x))|x\in X\rangle\}$ where, T, I, and F are the truth, indeterminacy, and falsity membership functions, such that $T, I, F: X \longrightarrow [0, 1]$ and the sum of T, I, and F is unrestricted, $0 \le T_N + I_N + F_N \le 3^+$.

Definition 4 Single-Valued Trapezoidal Neutrosophic Number [13]: Let $t, i, f \in [0, 1]$ and $\alpha \le \beta \le \gamma \le \delta \in R$. Then a Single-Valued Trapezoidal Neutrosophic Number, denoted by, $\tilde{N}$ is defined as: $\tilde{N}=\langle(\alpha,\beta,\gamma,\delta); t, i, f\rangle$ with the truth, indeterminacy, and falsity membership functions defined as:

$$T_{\tilde{N}}(x)=\begin{cases}\frac{x-\alpha}{\beta-\alpha}\cdot t_{\tilde{N}} & \alpha\le x\le\beta\\ t_{\tilde{N}} & \beta\le x\le\gamma\\ \frac{\delta-x}{\delta-\gamma}\cdot t_{\tilde{N}} & \gamma\le x\le\delta\\ 0 & \text{otherwise}\end{cases}$$

$$I_{\tilde{N}}(x)=\begin{cases}\frac{(\beta-x+i_{\tilde{N}}(x-\alpha))}{\beta-\alpha} & \alpha\le x\le\beta\\ i_{\tilde{N}} & \beta\le x\le\gamma\\ \frac{(x-\gamma+i_{\tilde{N}}(\delta-x))}{\delta-\gamma} & \gamma\le x\le\delta\\ 1 & \text{otherwise}\end{cases}$$

$$F_{\tilde{N}}(x)=\begin{cases}\frac{(\beta-x+f_{\tilde{N}}(x-\alpha))}{\beta-\alpha} & \alpha\le x\le\beta\\ f_{\tilde{N}} & \beta\le x\le\gamma\\ \frac{(x-\gamma+f_{\tilde{N}}(\delta-x))}{\delta-\gamma} & \gamma\le x\le\delta\\ 1 & \text{otherwise}\end{cases}$$

Definition 5 Trapezoidal Neutrosophic Number [12]: A Trapezoidal Neutrosophic Number, denoted by, $Tr\bar{N}$ is defined as: $Tr\bar{N}=\langle(\alpha_T, \beta_T, \gamma_T, \delta_T), (\alpha_I, \beta_I, \gamma_I, \delta_I), (\alpha_F, \beta_F, \gamma_F, \delta_F)\rangle$. If $\beta_T=\gamma_T$, $\beta_I=\gamma_I$, and $\beta_F=\gamma_F$, then it becomes a Triangular Neutrosophic Number.

Let there be two Trapezoidal Neutrosophic Numbers be: $Tr\bar{N}_1=\langle(\alpha_{T1}, \beta_{T1}, \gamma_{T1}, \delta_{T1}), (\alpha_{I1}, \beta_{I1}, \gamma_{I1}, \delta_{I1}), (\alpha_{F1}, \beta_{F1}, \gamma_{F1}, \delta_{F1})\rangle = \langle(0.9, 1.0, 1.0, 1.0), (0.1, 0.2, 0.3, 0.4), (0.0, 0.0, 0.1, 0.2)\rangle$, $Tr\bar{N}_2= \langle(\alpha_{T2}, \beta_{T2}, \gamma_{T2}, \delta_{T2}), (\alpha_{I2}, \beta_{I2}, \gamma_{I2}, \delta_{I2}), (\alpha_{F2}, \beta_{F2}, \gamma_{F2}, \delta_{F2})\rangle=\langle(0.8, 0.9, 1.0, 1.0), (0.2, 0.3, 0.4, 0.5), (0.0, 0.1, 0.2, 0.3)\rangle$, and $\lambda=2$.

Definition 6 Operation of Trapezoidal Neutrosophic Numbers [12]: Let there be two Trapezoidal Neutrosophic Numbers be: $Tr\bar{N}_1$, $Tr\bar{N}_2$, and $\lambda=2$. Then, the following operational rules hold:

- Addition ($\oplus$):

$$Tr\bar{N}_1\oplus Tr\bar{N}_2=\left\langle\begin{matrix}\begin{pmatrix}\alpha_{T1}+\alpha_{T2}-\alpha_{T1}\alpha_{T2}, \beta_{T1}+\beta_{T2}-\beta_{T1}\beta_{T2},\\ \gamma_{T1}+\gamma_{T2}-\gamma_{T1}\gamma_{T2}, \delta_{T1}+\delta_{T2}-\delta_{T1}\delta_{T2}\end{pmatrix},\\ (\alpha_{I1}\alpha_{I2}, \beta_{I1}\beta_{I2}, \gamma_{I1}\gamma_{I2}, \delta_{I1}\delta_{I2}),\\ (\alpha_{F1}\alpha_{F2}, \beta_{F1}\beta_{F2}, \gamma_{F1}\gamma_{F2}, \delta_{F1}\delta_{F2})\end{matrix}\right\rangle$$

- Multiplication ($\otimes$):

$$Tr\bar{N}_1\otimes Tr\bar{N}_2=\left\langle\begin{matrix}(\alpha_{T1}\alpha_{T2}, \beta_{T1}\beta_{T2}, \gamma_{T1}\gamma_{T2}, \delta_{T1}\delta_{T2}),\\ \begin{pmatrix}\alpha_{I1}+\alpha_{I2}-\alpha_{I1}\alpha_{I2}, \beta_{I1}+\beta_{I2}-\beta_{I1}\beta_{I2}\\ \gamma_{I1}+\gamma_{I2}-\gamma_{I1}\gamma_{I2}, \delta_{I1}+\delta_{I2}-\delta_{I1}\delta_{I2}\end{pmatrix}\\ \begin{pmatrix}\alpha_{F1}+\alpha_{F2}-\alpha_{F1}\alpha_{F2}, \beta_{F1}+\beta_{F2}-\beta_{F1}\beta_{F2}\\ \gamma_{F1}+\gamma_{F2}-\gamma_{F1}\gamma_{F2}, \delta_{F1}+\delta_{F2}-\delta_{F1}\delta_{F2}\end{pmatrix}\end{matrix}\right\rangle$$

- Scalar Multiplication (for $\lambda > 0$):

$$\lambda(Tr\bar{N}_1)=\left\langle\begin{matrix}\begin{pmatrix}1-(1-\alpha_{T1})^\lambda, 1-(1-\beta_{T1})^\lambda,\\ 1-(1-\gamma_{T1})^\lambda, 1-(1-\delta_{T1})^\lambda\end{pmatrix},\\ (\alpha_{I1}^\lambda, \beta_{I1}^\lambda, \gamma_{I1}^\lambda, \delta_{I1}^\lambda),\\ (\alpha_{F1}^\lambda, \beta_{F1}^\lambda, \gamma_{F1}^\lambda, \delta_{F1}^\lambda)\end{matrix}\right\rangle$$

- Power (for $\lambda > 0$):

$$Tr\bar{N}_1^{\lambda}=\left\langle\begin{matrix}(\alpha_{T1}^\lambda, \beta_{T1}^\lambda, \gamma_{T1}^\lambda, \delta_{T1}^\lambda),\\ \begin{pmatrix}1-(1-\alpha_{I1})^\lambda, 1-(1-\beta_{I1})^\lambda,\\ 1-(1-\gamma_{I1})^\lambda, 1-(1-\delta_{I1})^\lambda\end{pmatrix},\\ \begin{pmatrix}1-(1-\alpha_{F1})^\lambda, 1-(1-\beta_{F1})^\lambda\\ 1-(1-\gamma_{F1})^\lambda, 1-(1-\delta_{F1})^\lambda,\end{pmatrix}\end{matrix}\right\rangle$$

Working examples Definition 6:

- Addition:

$$Tr\bar{N}_1\oplus Tr\bar{N}_2$$
$$=\left\langle\begin{matrix}\begin{pmatrix}0.9+0.8-(0.9)(0.8), 1.0+0.9-(1.0)(0.9),\\ 1.0+1.0-(1.0)(1.0), 1.0+1.0-(1.0)(1.0)\end{pmatrix},\\ ((0.1)(0.2), (0.2)(0.3), (0.3)(0.4), (0.4)(0.5)),\\ ((0.0)(0.0), (0.0)(0.1), (0.1)(0.2), (0.2)(0.3))\end{matrix}\right\rangle$$
$$=\left\langle\begin{matrix}(0.98, 1.00, 1.00, 1.00),\\ (0.02, 0.06, 0.12, 0.20),\\ (0.00, 0.00, 0.02, 0.06)\end{matrix}\right\rangle$$

- Multiplication

$$Tr\bar{N}_1\oplus Tr\bar{N}_2$$
$$=\left\langle\begin{matrix}((0.9)(0.8), (1.0)(0.9), (1.0)(1.0), (1.0)(1.0)),\\ \begin{pmatrix}(0.1)+(0.2)-(0.1)(0.2), (0.2)+(0.3)-(0.2)(0.3),\\ (0.3)+(0.4)-(0.3)(0.4), (0.4)+(0.5)-(0.4)(0.5)\end{pmatrix},\\ \begin{pmatrix}(0.0)+(0.0)-(0.0)(0.0), (0.0)+(0.1)-(0.0)(0.1),\\ (0.1)+(0.2)-(0.1)(0.2), (0.2)+(0.3)-(0.2)(0.3)\end{pmatrix}\end{matrix}\right\rangle$$
$$=\left\langle\begin{matrix}(0.72, 0.90, 1.00, 1.00),\\ (0.28, 0.44, 0.58, 0.70),\\ (0.00, 0.00, 0.28, 0.44)\end{matrix}\right\rangle$$

- Scalar Multiplication

$$2(Tr\bar{N}_1)=\left\langle\begin{matrix}\begin{pmatrix}1-(1-0.9)^2, 1-(1-1.0)^2,\\ 1-(1-1.0)^2, 1-(1-1.0)^2\end{pmatrix},\\ (0.1^2, 0.2^2, 0.3^2, 0.4^2),\\ (0.0^2, 0.1^2, 0.2^2, 0.3^2)\end{matrix}\right\rangle$$

$$= \langle \begin{matrix} (0.99,\ 1.00,\ 1.00,\ 1.00), \\ (0.01,\ 0.04,\ 0.09,\ 0.16), \\ (0.00,\ 0.01,\ 0.04,\ 0.09) \end{matrix} \rangle$$

- Power

$$Tr\overline{N}_1^{\,2} = \langle \begin{matrix} (0.9^2,\ 1.0^2,\ 1.0^2,\ 1.0^2), \\ \begin{pmatrix} 1 - (1 - 0.1)^2,\ 1 - (1 - 0.2)^2, \\ 1 - (1 - 0.3)^2,\ 1 - (1 - 0.4)^2 \end{pmatrix}, \\ \begin{pmatrix} 1 - (1 - 0.0)^2,\ 1 - (1 - 0.1)^2 \\ 1 - (1 - 0.2)^2,\ 1 - (1 - 0.3)^2, \end{pmatrix} \end{matrix} \rangle$$

$$= \langle \begin{matrix} (0.81,\ 1.00,\ 1.00,\ 1.00), \\ (0.01,\ 0.36,\ 0.64,\ 0.19), \\ (0.00,\ 0.01,\ 0.36,\ 0.51) \end{matrix} \rangle$$

2.3 Defuzzification Method

In this section, the score function will be used to defuzzify the Trapezoidal Neutrosophic Number.

Definition 7 Score Function of Trapezoidal Neutrosophic Number [12]: Let $Tr\overline{N}$ be a Trapezoidal Neutrosophic Number, $Tr\overline{N} = \langle (\alpha_T, \beta_T, \gamma_T, \delta_T), (\alpha_I, \beta_I, \gamma_I, \delta_I), (\alpha_F, \beta_F, \gamma_F, \delta_F) \rangle$. Then, the score function of a Trapezoidal Neutrosophic Number, denoted as $S(Tr\overline{N})$, is defined as: $S(Tr\overline{N}) = \frac{1}{3}\left(2 + \frac{\alpha_T + \beta_T + \gamma_T + \delta_T}{4} - \frac{\alpha_I + \beta_I + \gamma_I + \delta_I}{4} - \frac{\alpha_F + \beta_F + \gamma_F + \delta_F}{4}\right)$ such that, $S(Tr\overline{N}) \in [0,1]$.

Working examples Definition 7:

$$S(Tr\overline{N}_1) = \frac{1}{3}\left(2 + \frac{0.9+1.0+1.0+1.0}{4} - \frac{0.1+0.2+0.3+0.4}{4} - \frac{0.0+0.0+0.1+0.2}{4}\right) = 0.8833$$

2.4 Aggregation Method

Based on the operation of Trapezoidal Neutrosophic Numbers in Definition 6, two aggregation methods to combine the information are employed: the weighted arithmetic and the weighted geometric aggregation method. The weighted arithmetic aggregation method is adopted for its ability to preserve the overall average while ensuring that each criterion's contribution is proportional to its assigned weight. The weighted geometric aggregation method, on the other hand, is well-suited to reflect multiplicative interaction effects among criteria, ensuring that very low or high values exert their expected influence. Using both methods provides a more robust aggregation framework that offers complementary perspectives.

Definition 8 Trapezoidal Neutrosophic Number Weighted Arithmetic Averaging Operators [12]: Let $Tr\overline{N}_p = \langle (\alpha_{T_p}, \beta_{T_p}, \gamma_{T_p}, \delta_{T_p}), (\alpha_{I_p}, \beta_{I_p}, \gamma_{I_p}, \delta_{I_p}), (\alpha_{F_p}, \beta_{F_p}, \gamma_{F_p}, \delta_{F_p}) \rangle$, (p=1, 2,…, q) be a collection of Trapezoidal Neutrosophic Numbers. Then, the Weighted Arithmetic Averaging operators (TNNWAA) are defined as:

$$TNNWAA(Tr\overline{N}_1, Tr\overline{N}_2, \ldots, Tr\overline{N}_q) = \bigotimes_{p=1}^{q} (w_p Tr\overline{N}_q)$$

$$= \langle \begin{matrix} \begin{pmatrix} 1 - \prod_{p=1}^{q}\left(1 - \alpha_{T_p}\right)^{w_p},\ 1 - \prod_{p=1}^{q}\left(1 - \beta_{T_p}\right)^{w_p}, \\ 1 - \prod_{p=1}^{q}\left(1 - \gamma_{T_p}\right)^{w_p},\ 1 - \prod_{p=1}^{q}\left(1 - \delta_{T_p}\right)^{w_p} \end{pmatrix}, \\ \begin{pmatrix} \prod_{p=1}^{q} \alpha_{I_p}^{\ w_p},\ \prod_{p=1}^{q} \beta_{I_p}^{\ w_p}, \\ \prod_{p=1}^{q} \gamma_{I_p}^{\ w_p},\ \prod_{p=1}^{q} \delta_{I_p}^{\ w_p} \end{pmatrix}, \\ \begin{pmatrix} \prod_{p=1}^{q} \alpha_{F_p}^{\ w_p},\ \prod_{p=1}^{q} \beta_{F_p}^{\ w_p}, \\ \prod_{p=1}^{q} \gamma_{I_p}^{\ w_p},\ \prod_{p=1}^{q} \delta_{F_p}^{\ w_p} \end{pmatrix} \end{matrix} \rangle$$

where, $w_p \in [0,1]$ and $\sum_{p=1}^{q} w_p = 1$, w_p representing the weight of the p^{th} Trapezoidal Neutrosophic Number.

Working examples Definition 8: Let w_1=0.55 and w_2=0.45,

$$TNNWAA(Tr\overline{N}_1, Tr\overline{N}_2) = \bigotimes_{p=1}^{2} (w_p Tr\overline{N}_q)$$

$$= \langle \begin{matrix} \begin{pmatrix} 1-(1 - 0.9)^{0.55}+1 - (1 - 0.8)^{0.45}, \\ 1 - (1 - 1.0)^{0.55}+1 - (1 - 0.9)^{0.45}, \\ 1 - (1 - 1.0)^{0.55}+1 - (1 - 1.0)^{0.45}, \\ 1 - (1 - 1.0)^{0.55}+1 - (1 - 1.0)^{0.45} \end{pmatrix}, \\ \begin{pmatrix} 0.1^{0.55}+ 0.2^{0.45},\ 0.2^{0.55}+ 0.3^{0.45}, \\ 0.3^{0.55}+ 0.4^{0.45},\ 0.4^{0.55}+ 05^{0.45} \end{pmatrix}, \\ \begin{pmatrix} 0.0^{0.55}+ 0.0^{0.45},\ 0.0^{0.55}+ 0.1^{0.45}, \\ 0.1^{0.55}+ 0.2^{0.45},\ 0.2^{0.55}+ 0.3^{0.45} \end{pmatrix} \end{matrix} \rangle$$

$$= \langle \begin{matrix} (1.2335,\ 1.6452,\ 2.0000,\ 2.0000), \\ (0.7665,\ 0.9943,\ 1.1778,\ 1.3362), \\ (0.0000,\ 0.3548,\ 0.7665,\ 0.9943) \end{matrix} \rangle$$

Definition 9 Trapezoidal Neutrosophic Number Weighted Geometric Averaging Operators [12]: Let $Tr\overline{N}_p = \langle (\alpha_{T_p}, \beta_{T_p}, \gamma_{T_p}, \delta_{T_p}), (\alpha_{I_p}, \beta_{I_p}, \gamma_{I_p}, \delta_{I_p}), (\alpha_{F_p}, \beta_{F_p}, \gamma_{F_p}, \delta_{F_p}) \rangle$, (p=1, 2,…, q) be a collection of Trapezoidal Neutrosophic Numbers. Then, the Weighted Geometric Averaging operators (TNNWGA) are defined as:

$$TNNWGA(Tr\overline{N}_1, Tr\overline{N}_2, \ldots, Tr\overline{N}_q) = \bigotimes_{p=1}^{q} (Tr\overline{N}_q^{\ w_p})$$

$$= \langle \begin{matrix} \begin{pmatrix} \prod_{p=1}^{q} \alpha_{T_p}^{\ w_p},\ \prod_{p=1}^{q} \beta_{T_p}^{\ w_p}, \\ \prod_{p=1}^{q} \gamma_{T_p}^{\ w_p},\ \prod_{p=1}^{q} \delta_{T_p}^{\ w_p} \end{pmatrix}, \\ \begin{pmatrix} 1 - \prod_{p=1}^{q}\left(1 - \alpha_{I_p}\right)^{w_p},\ 1 - \prod_{p=1}^{q}\left(1 - \beta_{I_p}\right)^{w_p} \\ 1 - \prod_{p=1}^{q}\left(1 - \gamma_{I_p}\right)^{w_p},\ 1 - \prod_{p=1}^{q}\left(1 - \delta_{I_p}\right)^{w_p} \end{pmatrix}, \\ \begin{pmatrix} 1 - \prod_{p=1}^{q}\left(1 - \alpha_{F_p}\right)^{w_p},\ 1 - \prod_{p=1}^{q}\left(1 - \beta_{F_p}\right)^{w_p} \\ 1 - \prod_{p=1}^{q}\left(1 - \gamma_{F_p}\right)^{w_p},\ 1 - \prod_{p=1}^{q}\left(1 - \delta_{F_p}\right)^{w_p} \end{pmatrix} \end{matrix} \rangle$$

where, $w_p \in [0,1]$ and $\sum_{p=1}^{q} w_p = 1$, w_p representing the weight of the p^{th} Trapezoidal Neutrosophic Number.

Working examples Definition 9: let w_1=0.45 and w_2=0.55:

$$TNNWGA(Tr\bar{N}_1, Tr\bar{N}_2) = \bigotimes_{p=1}^{q} \left(Tr\bar{N}_q^{\,w_p}\right)$$

$$= \left\langle \begin{pmatrix} 0.9^{0.45} + 0.8^{0.55}, 1.0^{0.45} + 0.9^{0.55}, \\ 1.0^{0.45} + 1.0^{0.55}, 1.0^{0.45} + 1.0^{0.55} \end{pmatrix}, \begin{pmatrix} 1\text{-}(1\text{-}0.1)^{0.45} + 1\text{-}(1\text{-}0.2)^{0.55}, \\ 1\text{-}(1\text{-}0.2)^{0.45} + 1\text{-}(1\text{-}0.3)^{0.55}, \\ 1\text{-}(1\text{-}0.3)^{0.45} + 1\text{-}(1\text{-}0.4)^{0.55}, \\ 1\text{-}(1\text{-}0.4)^{0.45} + 1\text{-}(1\text{-}0.5)^{0.55} \end{pmatrix}, \begin{pmatrix} 1\text{-}(1\text{-}0.0)^{0.45} + 1\text{-}(1\text{-}0.0)^{0.55}, \\ 1\text{-}(1\text{-}0.0)^{0.45} + 1\text{-}(1\text{-}0.1)^{0.55}, \\ 1\text{-}(1\text{-}0.1)^{0.45} + 1\text{-}(1\text{-}0.2)^{0.55}, \\ 1\text{-}(1\text{-}0.2)^{0.45} + 1\text{-}(1\text{-}0.3)^{0.55} \end{pmatrix} \right\rangle$$

$$= \left\langle \begin{matrix} (1.8382, 1.9437, 2.0000, 2.0000), \\ (0.1618, 0.2737, 0.3932, 0.5223), \\ (0.0000, 0.0563, 0.1618, 0.2737) \end{matrix} \right\rangle$$

3. PROPOSED METHODOLOGY

This section introduces the proposed Multi-Criteria Decision Making (MCDM) Method based on Trapezoidal Neutrosophic Numbers (TrNNs) adopted from Reference [12] to solve complex decision-making problems.

To provide a clear understanding of our approach, we begin with an overview of the four-step process (Fig. 1). This methodology ensures systematic evaluation and comparison of alternatives under uncertainty. First, the linguistic decision-making matrix is constructed using TrNNs for each criterion relative to the alternatives. Second, the overall collective for each alternative is computed using weighted arithmetic and geometric aggregation methods. Third, a score function is applied to calculate the scores for each alternative. Finally, the alternatives are ranked by their calculated scores, allowing decision-makers to identify the most suitable choice.

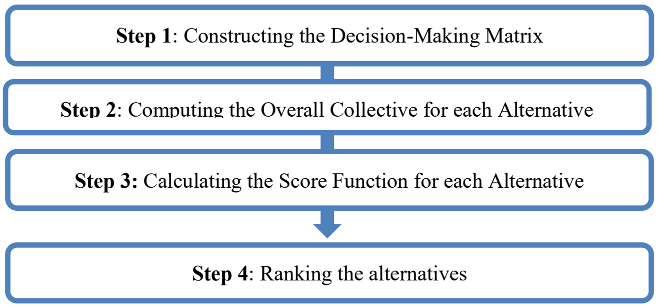

Fig. 1.A step-by-step process of the MCDM based on TrNNs

Suppose a multi-criteria decision-making problem consists of a set of alternatives, $\bar{A}_i = (\bar{A}_1, \bar{A}_2, \ldots, \bar{A}_m)$, and a set of criteria $\bar{C}_j = (\bar{C}_1, \bar{C}_2, \ldots, \bar{C}_n)$. The decision matrix is denoted as $\bar{N}_{ij} = \langle (\alpha_{T_{ij}}, \beta_{T_{ij}}, \gamma_{T_{ij}}, \delta_{T_{ij}}), (\alpha_{I_{ij}}, \beta_{I_{ij}}, \gamma_{I_{ij}}, \delta_{I_{ij}}), (\alpha_{F_{ij}}, \beta_{F_{ij}}, \gamma_{F_{ij}}, \delta_{F_{ij}}) \rangle$, where $\bar{N}_{ij}$ represents a TrNN corresponding to the j^{th} criterion $\bar{C}_j$ for the i^{th} alternative $\bar{A}_i$, where $1 \le i \le m$, $1 \le j \le n$. The weights for the criteria are also provided as TrNNs. The weight vector is denoted as $W = (w_1, w_2, \ldots, w_n)$, where each $\bar{w}$ is a TrNN that represents the weight of the j^{th} criterion $\bar{C}_j$.

Step 1: Constructing the Decision-Making Matrix

The decision-maker assesses each alternative $\bar{A}_i$, $1 \le i \le m, 1 \le j \le n$ against the j^{th} criteria $\bar{C}_j$ to construct the linguistic decision-making matrix, $[\bar{N}_{ij}]_{m\times n}$ using the assessment scale in Table 1. Note that the linguistic terms in Table 1 were converted into TrNNs based on expert opinions and supported by values from earlier research [12]. The linguistic decision-matrix will be converted into its respective TrNNs. For example, the decision-maker rates Alternative 1, $\bar{A}_1$, with respect to Criterion 1, $\bar{C}_1$, as "High (H)". Thus, the corresponding TrNNs are $\bar{N}_{11} = \langle(0.8, 0.9, 1.0, 1.0), (0.2, 0.3, 0.4, 0.5), (0.0, 0.1, 0.2, 0.3)\rangle$. Similarly, when the decision-maker rates $\bar{A}_1$, with respect to Criterion 3, $\bar{C}_3$ as "Near Optimal (NO)", then, the corresponding TrNNs are $\bar{N}_{13} = \langle(0.7, 0.8, 0.9, 1.0), (0.3, 0.4, 0.5, 0.6), (0.1, 0.2, 0.3, 0.4)\rangle$.

Table 1. The Assessment Scale of the Decision Matrix

Linguistic Variable (Criteria 1 and 2)	Linguistic Variable (Criteria 3)	Trapezoidal Neutrosophic Numbers
Absolutely High (AH)	Absolutely Optimal (AO)	⟨(0.9, 1.0, 1.0, 1.0), (0.1, 0.2, 0.3, 0.4), (0.0, 0.0, 0.1, 0.2)⟩
High (H)	Optimal (O)	⟨(0.8, 0.9, 1.0, 1.0), (0.2, 0.3, 0.4, 0.5), (0.0, 0.1, 0.2, 0.3)⟩
Fairy High (FH)	Near Optimal (NO)	⟨(0.7, 0.8, 0.9, 1.0), (0.3, 0.4, 0.5, 0.6), (0.1, 0.2, 0.3, 0.4)⟩
Medium (M)	Moderately Optimal (MO)	⟨(0.4, 0.7, 0.8, 0.9), (0.4, 0.5, 0.6, 0.7), (0.2, 0.3, 0.4, 0.5)⟩
Fairly Low (FL)	Sub-Optimal (SO)	⟨(0.1, 0.4, 0.7, 0.8), (0.5, 0.6, 0.7, 0.8), (0.3, 0.4, 0.5, 0.6)⟩
Low (L)	Fairly Suboptimal (FSO)	⟨(0.0, 0.1, 0.4, 0.7), (0.6, 0.7, 0.8, 0.9), (0.4, 0.5, 0.6, 0.7)⟩
Absolutely Low (AL)	Absolutely Sub-Optimal (ASO)	⟨(0.0, 0.0, 0.1, 0.2), (0.7, 0.8, 0.9, 1.0), (0.5, 0.6, 0.7, 0.8)⟩

Step 2: Computing the Overall Collective for each Alternative

Based on Definitions 8 and 9, the overall collective for each alternative can be computed by utilizing the $TNNWAA(\bar{A}_i)$ and $TNNWGA(\bar{A}_i)$ as follows:

$$= TNNWAA((\bar{N}_{11}, \bar{N}_{12}, \ldots, \bar{N}_{ij})$$

$$= w_1\bar{N}_{11} \otimes w_2\bar{N}_{12} \otimes \cdots \otimes w_n\bar{N}_{mn} = \bigotimes_{j=1}^{n} w_j\bar{N}_{ij}$$

$$= \left\langle \begin{pmatrix} 1 - \prod_{j=1}^{n}\left(1 - \alpha_{T_{ij}}\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - \beta_{T_{ij}}\right)^{w_j}, \\ 1 - \prod_{j=1}^{n}\left(1 - \gamma_{T_{ij}}\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - \delta_{T_{ij}}\right)^{w_j} \end{pmatrix}, \begin{matrix} \left(\prod_{j=1}^{n} \alpha_{I_{ij}}^{\,w_j}, \prod_{j=1}^{n} \beta_{I_{ij}}^{\,w_j}, \prod_{j=1}^{n} \gamma_{I_{ij}}^{\,w_j}, \prod_{j=1}^{n} \delta_{I_{ij}}^{\,w_j}\right), \\ \left(\prod_{j=1}^{n} \alpha_{F_{ij}}^{\,w_j}, \prod_{j=1}^{n} \beta_{F_{ij}}^{\,w_j}, \prod_{j=1}^{n} \gamma_{F_{ij}}^{\,w_j}, \prod_{j=1}^{n} \delta_{F_{ij}}^{\,w_j}\right) \end{matrix} \right\rangle \quad (1)$$

$$
\begin{aligned}
&= \mathrm{TNNWGA}\left(\bar{N}_{11}, \bar{N}_{12}, \ldots, \bar{N}_{ij}\right) \\
&= \bar{N}_{11}^{\,w_1} \otimes \bar{N}_{12}^{\,w_2} \otimes \cdots \otimes \bar{N}_{mn}^{\,w_n} = \bigotimes_{j=1}^{n} \bar{N}_{ij}^{\,w_j} \\
&= \left\langle
\begin{array}{l}
\left(\prod_{j=1}^{n} \alpha_{T_{ij}}^{\,w_j}, \prod_{j=1}^{n} \beta_{T_{ij}}^{\,w_j}, \prod_{j=1}^{n} \gamma_{T_{ij}}^{\,w_j}, \prod_{j=1}^{n} \delta_{T_{ij}}^{\,w_j}\right), \\
\begin{pmatrix} 1-\prod_{j=1}^{n}\left(1-\alpha_{I_{ij}}\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-\beta_{I_{ij}}\right)^{w_j}, \\ 1-\prod_{j=1}^{n}\left(1-\gamma_{I_{ij}}\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-\delta_{I_{ij}}\right)^{w_j} \end{pmatrix}, \\
\begin{pmatrix} 1-\prod_{j=1}^{n}\left(1-\alpha_{F_{ij}}\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-\beta_{F_{ij}}\right)^{w_j}, \\ 1-\prod_{j=1}^{n}\left(1-\gamma_{F_{ij}}\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-\delta_{F_{ij}}\right)^{w_j} \end{pmatrix}
\end{array}
\right\rangle
\end{aligned}
\tag{2}
$$

where, $\bar{N}_{ij} = \langle(\alpha_{T_{ij}}, \beta_{T_{ij}}, \gamma_{T_{ij}}, \delta_{T_{ij}}), (\alpha_{I_{ij}}, \beta_{I_{ij}}, \gamma_{I_{ij}}, \delta_{I_{ij}}), (\alpha_{F_{ij}}, \beta_{F_{ij}}, \gamma_{F_{ij}}, \delta_{F_{ij}})\rangle$, $w_j (j=1, 2,\ldots, n)$ is the weight of the j^{th} criterion $\bar{C}_j$, and $\sum_{j=1}^{n} w_j = 1$.

Step 3: Calculating the Score Function for each Alternative

The score function of the collective overall for each alternative, $S(\bar{A}_i)$, such that $S(\bar{A}_i) \in [0, 1]$ will be calculated by using Definition 7 as follows:

$$S(\bar{A}_i) = \frac{1}{3}\left(2 + \frac{\alpha_T + \beta_T + \gamma_T + \delta_T}{4} - \frac{\alpha_I + \beta_I + \gamma_I + \delta_I}{4} - \frac{\alpha_F + \beta_F + \gamma_F + \delta_F}{4}\right) \tag{3}$$

Step 4: Ranking the alternatives

The alternatives are ranked based on the score function values computed in Step 3. The alternative with the highest score indicates the most suitable choice.

4. Result and Discussion

This section is organized into three sub-sections: Section A presents the Results of the Biogas Recovery Selection, Section B discusses the Sensitivity Analysis, and Section C provides the Comparative Analysis.

A. Results: Biogas Recovery Selection Process

The proposed methodology is applied to select the most suitable substrates for anaerobic digestion in biogas recovery. The process considers five substrates: Carbohydrate-Rich Food Waste (CRFW), Fiber-Rich Food Waste (FRFW), Protein-Rich Food Waste (PRFW), Palm Oil Mill Effluent Sludge (POMES), and Palm Oil Mill Effluent (POME). As an initial assumption, POMES is identified as the most suitable. This is explained by its concentrated organic content, especially in VS, which provides a higher energy yield [37]. Moreover, POMES exhibits the highest buffering capacity, thereby enhancing stability. In contrast with other substrates that contain more lignocellulosic material, then to cause the rapid acidification, while POME is more diluted organic load per unit volume [32], [38]. These compositional and stability differences account for POMES's superior performance in biogas recovery.

To address the substrate selection problem, let the set of alternatives (substrates) be denoted by, $\bar{A}_i = (\bar{A}_1, \bar{A}_2, \ldots, \bar{A}_m)$, where: $\bar{A}_1$=CRFW, $\bar{A}_2$=FRFW, $\bar{A}_3$=PRFW, $\bar{A}_4$=POMES, and $\bar{A}_5$=POME. These alternatives are evaluated based on three decision criteria (properties), denoted by, $\bar{C}_i = (\bar{C}_1, \bar{C}_2, \ldots, \bar{C}_n)$, where $\bar{C}_1$=TS, $\bar{C}_2$=VS, and $\bar{C}_3$=PH. Assume that the importance of each criterion is represented by the weight vector, W=(0.37, 0.47, 0.16), then the evaluation of each substrate with respect to the three criteria is performed by the decision-maker under the TrNNs environment, which can be described as follows:

Step 1: Constructing the Decision-Making Matrix

The decision-maker assessed each alternative against the criteria to form the linguistic decision-making matrix, using the assessment scale in Table 1. The linguistic decision matrix and its corresponding TrNNs are presented in Table 2. As an illustration, the decision-maker rates Alternative 1, $\bar{A}_1$, with respect to Criterion 1, $\bar{C}_1$, as "Fairly Low (FL)". Thus, the corresponding TrNNs are $\bar{N}_{11}$=⟨(0.1, 0.4, 0.7, 0.8), (0.5, 0.6, 0.7, 0.8),(0.3, 0.4, 0.5, 0.6)⟩.

Table 2. The linguistic Decision-Making Matrix and the corresponding Trapezoidal Neutrosophic Numbers

Alternative ($\bar{A}_i$)	Criteria ($\bar{C}_j$)		
	$\bar{C}_1$	$\bar{C}_2$	$\bar{C}_3$
$\bar{A}_1$	$FL = \bar{N}_{11}$ (0.1, 0.4, 0.7, 0.8), = ⟨(0.5, 0.6, 0.7, 0.8), (0.3, 0.4, 0.5, 0.6)	$H = \bar{N}_{12}$ (0.8, 0.9, 1.0, 1.0), = ⟨(0.2, 0.3, 0.4, 0.5), (0.0, 0.1, 0.2, 0.3)	$ASO = \bar{N}_{13}$ (0.0, 0.0, 0.1, 0.2), = ⟨(0.7, 0.8, 0.9, 1.0), (0.5, 0.6, 0.7, 0.8)
$\bar{A}_2$	$L = \bar{N}_{21}$ (0.0, 0.1, 0.4, 0.7), =⟨(0.6, 0.7, 0.8, 0.9),⟩ (0.4, 0.5, 0.6, 0.7)	$FH = \bar{N}_{22}$ (0.7, 0.8, 0.9, 1.0), =⟨(0.3, 0.4, 0.5, 0.6),⟩ (0.1, 0.2, 0.3, 0.4)	$ASO = \bar{N}_{13}$ (0.0, 0.0, 0.1, 0.2), = ⟨(0.7, 0.8, 0.9, 1.0), (0.5, 0.6, 0.7, 0.8)
$\bar{A}_3$	$L = \bar{N}_{31}$ (0.0, 0.1, 0.4, 0.7), = ⟨(0.6, 0.7, 0.8, 0.9), (0.4, 0.5, 0.6, 0.7)	$H = \bar{N}_{32}$ **(0.8, 0.9, 1.0, 1.** = ⟨**(0.2, 0.3, 0.4, 0.** **(0.0, 0.1, 0.2, 0.**	$ASO = \bar{N}_{33}$ (0.0, 0.0, 0.1, 0.2), = ⟨(0.7, 0.8, 0.9, 1.0), (0.5, 0.6, 0.7, 0.8)
$\bar{A}_4$	$AH = \bar{N}_{41}$ (**0**.9, 1.0, 1.0, 1.0), =⟨(0.1, 0.2, 0.3, 0.4),⟩ (0.0, 0.0, 0.1, 0.2)	$AH = \bar{N}_{42}$ (0.9, 1.0, 1.0, 1.0), =⟨(0.1, 0.2, 0.3, 0.4),⟩ (0.0, 0.0, 0.1, 0.2)	$SO = \bar{N}_{43}$ (0.1, 0.4, 0.7, 0.8), = ⟨(0.5, 0.6, 0.7, 0.8), (0.3, 0.4, 0.5, 0.6)
$\bar{A}_5$	$H = \bar{N}_{51}$ (0.8, 0.9, 1.0, 1.0), = ⟨(0.2, 0.3, 0.4, 0.5), (0.0, 0.1, 0.2, 0.3)	$H = \bar{N}_{52}$ (0.8, 0.9, 1.0, 1.0), = ⟨(0.2, 0.3, 0.4, 0.5), (0.0, 0.1, 0.2, 0.3)	$ASO = \bar{N}_{53}$ (0.0, 0.0, 0.1, 0.2), = ⟨(0.7, 0.8, 0.9, 1.0), (0.5, 0.6, 0.7, 0.8)

Step 2: Computing the Overall Collective for each Alternative

The overall collective for the alternative displayed in Table 3 is developed using Eqs. (1) and (2). As an illustration, the overall collective of Alternative 1, $\bar{A}_1$ is computed as follows,

$$=\text{TNNWAA}(\bar{A}_1)=(\bar{N}_{11}, \bar{N}_{12},\ldots, \bar{N}_{15})= \bigotimes_{j=1}^{5} w_j\bar{N}_{ij}$$

$$=\left\langle \begin{pmatrix} 1\text{-}(1\text{-}0.1)^{0.37}+ 1\text{-}(1\text{-}0.8)^{0.47}+1\text{-}(1\text{-}0.0)^{0.16} - \\ (1\text{-}(1\text{-}0.1)^{0.37})(1\text{-}(1\text{-}0.8)^{0.47})(1\text{-}(1\text{-}0.0)^{0.16}), \\ 1\text{-}(1\text{-}0.4)^{0.37}+ 1\text{-}(1\text{-}0.9)^{0.47}+1\text{-}(1\text{-}0.0)^{0.16} - \\ (1\text{-}(1\text{-}0.4)^{0.37})(1\text{-}(1\text{-}0.9)^{0.47})(1\text{-}(1\text{-}0.0)^{0.16}), \\ 1\text{-}(1\text{-}0.7)^{0.37}+ 1\text{-}(1\text{-}1.0)^{0.47}+1\text{-}(1\text{-}0.1)^{0.16} - \\ (1\text{-}(1\text{-}0.7)^{0.37})(1\text{-}(1\text{-}1.0)^{0.47})(1\text{-}(1\text{-}0.1)^{0.16}), \\ (1\text{-}(1\text{-}0.8)^{0.37})+(1\text{-}(1\text{-}1.0)^{0.47})+(1\text{-}(1\text{-}0.2)^{0.16}) - \\ (1\text{-}(1\text{-}0.8)^{0.37})(1\text{-}(1\text{-}1.0)^{0.47})(1\text{-}(1\text{-}0.2)^{0.16}) \end{pmatrix}, \right.$$

$$\begin{pmatrix} 0.5^{0.37}+0.2^{0.47}+0.7^{0.16}, 0.6^{0.37}+0.3^{0.47}+0.8^{0.16}, \\ 0.7^{0.37}+0.4^{0.47}+0.9^{0.16}, 0.8^{0.37}+0.5^{0.47}+1.0^{0.16} \end{pmatrix},$$

$$\left. \begin{pmatrix} 0.3^{0.37}+0.0^{0.47}+0.5^{0.16}, 0.4^{0.37}+0.1^{0.47}+0.6^{0.16}, \\ 0.5^{0.37}+0.2^{0.47}+0.7^{0.16}, 0.6^{0.37}+0.3^{0.47}+0.8^{0.16} \end{pmatrix} \right\rangle$$

$$= \left\langle \begin{matrix} (0.5689, 0.8334, 1.3702, 1.4680), \\ (2.1876, 2.3606, 2.5097, 2.6427), \\ (1.5355, 1.9728, 2.1876, 2.3606) \end{matrix} \right\rangle$$

$$=\text{TNNWGA}(\bar{A}_1)=(\bar{N}_{11}, \bar{N}_{12},\ldots, \bar{N}_{15})= \bigotimes_{j=1}^{5} \bar{N}_{ij}^{w_j}$$

$$=\left\langle \begin{pmatrix} 0.1^{0.37}+0.8^{0.47}+0.0^{0.16}, 0.4^{0.37}+0.9^{0.47}+0.0^{0.16}, \\ 0.7^{0.37}+1.0^{0.47}+0.1^{0.16}, 0.8^{0.37}+1.0^{0.47}+0.2^{0.16} \end{pmatrix}, \right.$$

$$\begin{pmatrix} 1\text{-}(1\text{-}0.5)^{0.37}+ 1\text{-}(1\text{-}0.2)^{0.47}+1\text{-}(1\text{-}0.7)^{0.16}\text{-} \\ (1\text{-}(1\text{-}0.5)^{0.37})(1\text{-}(1\text{-}0.2)^{0.47})(1\text{-}(1\text{-}0.7)^{0.16}), \\ 1\text{-}(1\text{-}0.5)^{0.37}+ 1\text{-}(1\text{-}0.2)^{0.47}+1\text{-}(1\text{-}0.7)^{0.16}\text{-} \\ (1\text{-}(1\text{-}0.5)^{0.37})(1\text{-}(1\text{-}0.2)^{0.47})(1\text{-}(1\text{-}0.7)^{0.16}), \\ 1\text{-}(1\text{-}0.5)^{0.37}+ 1\text{-}(1\text{-}0.2)^{0.47}+1\text{-}(1\text{-}0.7)^{0.16}\text{-} \\ (1\text{-}(1\text{-}0.5)^{0.37})(1\text{-}(1\text{-}0.2)^{0.47})(1\text{-}(1\text{-}0.7)^{0.16}), \\ (1\text{-}(1\text{-}0.8)^{0.37})+(1\text{-}(1\text{-}0.5)^{0.47})+(1\text{-}(1\text{-}1.0)^{0.16})\text{-} \\ (1\text{-}(1\text{-}0.8)^{0.37})(1\text{-}(1\text{-}0.5)^{0.47})(1\text{-}(1\text{-}1.0)^{0.16}) \end{pmatrix},$$

$$\left. \begin{pmatrix} 1\text{-}(1\text{-}0.3)^{0.37}+ 1\text{-}(1\text{-}0.0)^{0.47}+1\text{-}(1\text{-}0.5)^{0.16}\text{-} \\ (1\text{-}(1\text{-}0.3)^{0.37})(1\text{-}(1\text{-}0.0)^{0.47})(1\text{-}(1\text{-}0.5)^{0.16}), \\ 1\text{-}(1\text{-}0.4)^{0.37}+ 1\text{-}(1\text{-}0.1)^{0.47}+1\text{-}(1\text{-}0.6)^{0.16}\text{-} \\ (1\text{-}(1\text{-}0.4)^{0.37})(1\text{-}(1\text{-}0.1)^{0.47})(1\text{-}(1\text{-}0.6)^{0.16}), \\ 1\text{-}(1\text{-}0.5)^{0.37}+ 1\text{-}(1\text{-}0.2)^{0.47}+1\text{-}(1\text{-}0.7)^{0.16}\text{-} \\ (1\text{-}(1\text{-}0.5)^{0.37})(1\text{-}(1\text{-}0.2)^{0.47})(1\text{-}(1\text{-}0.7)^{0.16}), \\ (1\text{-}(1\text{-}0.6)^{0.37})+(1\text{-}(1\text{-}0.3)^{0.47})+(1\text{-}(1\text{-}0.8)^{0.16})\text{-} \\ (1\text{-}(1\text{-}0.6)^{0.37})(1\text{-}(1\text{-}0.3)^{0.47})(1\text{-}(1\text{-}0.8)^{0.16}) \end{pmatrix} \right\rangle$$

$$= \left\langle \begin{matrix} (1.3270, 1.6641, 2.5682, 2.6937), \\ (0.4971, 0.6588, 0.8574, 1.6020), \\ (0.2286, 0.3558, 0.4971, 0.6588) \end{matrix} \right\rangle$$

Table 3. The Overall Collective of the Alternative

$\bar{A}_i$	**TNNWAA($\bar{A}_i$)**	**TNNWGA($\bar{A}_i$)**
$\bar{A}_1$	(0.5689, 0.8334, 1.3702, 1.4680), ⟨(2.1876, 2.3606, 2.5097, 2.6427), (1.5355, 1.9728, 2.1876, 2.3606)	(1.3270, 1.6641, 2.5682, 2.6937), ⟨(0.4971, 0.6588, 0.8574, 1.6020), (0.2286, 0.3558, 0.4971, 0.6588)
$\bar{A}_2$	(0.4321, 0.5689, 0.8482, 1.3819), ⟨(2.3402, 2.4914, 2.6260, 2.7483), (1.9463, 2.1646, 2.3402, 2.4914)	(0.8457, 1.3270, 2.3560, 2.6493), ⟨(0.6093, 0.7825, 0.9965, 1.7227), (0.3246, 0.4591, 0.6093, 0.7825)
$\bar{A}_3$	(0.5307, 0.6994, 1.1861, 1.3819), ⟨(2.2416, 2.4092, 2.5541, 2.6837),⟩ (1.6075, 2.0341, 2.2416, 2.4092)	(0.9004, 1.3783, 2.4043, 2.6493), ⟨(0.5573, 0.7282, 0.9408, 1.6920), (0.2772, 0.4094, 0.5573, 0.7282)
$\bar{A}_4$	(1.2450, 2.0000, 2.0000, 2.0000), ⟨(1.6604, 1.9421, 2.1529, 2.3275),⟩ (0.8248, 0.8636, 1.6604, 1.9421)	(2.6053, 2.8636, 2.9445, 2.9649), ⟨(0.1913, 0.3141, 0.4498, 0.6043), (0.0555, 0.0785, 0.1913, 0.3141)
$\bar{A}_5$	**(0.9794, 1.2346, 2.0000, 2.0** **⟨(01.9652, 2.1733, 2.3458, 2.4** **(0.8950, 1.6869, 1.9652, 2.1**	**(1.8212, 1.9135, 2.6918, 2.77** **⟨(0.3526, 0.5007, 0.6825, 1.44** **(0.1050, 0.2227, 0.3526, 0.50**

Step 3: Calculating the Score Function for each Alternative

The score function for the collective overall for each alternative, $S(\bar{A}_i)$ calculated using Eq. (3), can be displayed in Table 5. As an illustration, the score function for Alternative 1, $S(\bar{A}_1)$ for each aggregation method, is obtained as follows:

Score Function for TNNWAA($\bar{A}_1$):

$$S(\bar{A}_1)=\frac{1}{3}\left(\frac{0.5689 + 0.8334 + 1.3702 + 1.4680}{4} - \frac{2.1876 + 2.3606 + 2.5097 + 2.6427}{4} - \frac{1.5355 + 1.9728 + 2.1876 + 2.3606}{4}\right)$$

$$= 0.4597$$

Score Function for TNNGAA($\bar{A}_1$):

$$S(\bar{A}_1)=\frac{1}{3}\left(\frac{1.3270 + 1.6641+2.5682 + 2.6937}{4} - \frac{0.4971 + 0.6588 + 0.8574 + 1.6020}{4} - \frac{0.2286 + 0.3558 + 0.4971 + 0.6588}{4}\right)$$

$$= 0.9081$$

Table 4. The Score Function of the Alternative

$\bar{A}_i$	**Score Function $(S(\bar{A}_i))$**	
	TNNWAA($\bar{A}_i$)	***TNNWGA($\bar{A}_i$)***
$\bar{A}_1$	$S(\bar{A}_1)$ = -0.4597	$S(\bar{A}_1)$ = 0.9081
$\bar{A}_2$	$S(\bar{A}_2)$ = -0.6598	$S(\bar{A}_2)$ = 0.7410
$\bar{A}_3$	$S(\bar{A}_3)$ = -0.5319	$S(\bar{A}_3)$ = 0.7868
$\bar{A}_4$	$S(\bar{A}_4)$ = 0.1559	$S(\bar{A}_4)$ = 1.4316
$\bar{A}_5$	$S(\bar{A}_5)$ = -0.1239	$S(\bar{A}_5)$ = 1.0868

Step 4: Ranking the alternatives

Based on the score functions calculated in Step 3, the ranking of the alternatives can be presented as follows:

Table 5. Ranking of the Alternatives

$\bar{A}_i$	**TNNWAA($\bar{A}_i$)**	**TNNWGA($\bar{A}_i$)**
$\bar{A}_1$	3	3
$\bar{A}_2$	5	5
$\bar{A}_3$	4	4
$\bar{A}_4$	1	1
$\bar{A}_5$	2	2

Note: $\bar{A}_1$=CRFW, $\bar{A}_2$=FRFW, $\bar{A}_3$=PRFW, $\bar{A}_4$=POMES, and $\bar{A}_5$=POME

Both rankings, whether using the arithmetic or geometric approach, produced the same ranking $\overline{A}_4 > \overline{A}_5 > \overline{A}_1 > \overline{A}_3 > \overline{A}_2$. This means the POMES is selected as the most suitable substrate, followed by POME, CRFW, PRFW, and FRFW.

B. Sensitivity analysis

To examine the robustness of the decision-making process, a sensitivity analysis was conducted by perturbing the input (linguistic terms) for Criterion 1 across all alternatives. The analysis was performed in two stages: a one-level-down adjustment (Table 6) and a one-level-up adjustment (Table 7). The results in Table 6 show that when the input is reduced by 1 level, the ranking of alternatives remains unchanged. Similarly, Table 7 presents the results of the one-level-up adjustment. Despite changes in the absolute score values, the ranking order remains identical. This stability suggests that the proposed methodology is not sensitive to downward or upward changes in Criterion 1, thereby supporting the robustness and consistency of the ranking results.

Table 6. The Sensitivity Analysis (One-Level-Down)

$\overline{A}_i$	TNNWAA($\overline{A}_i$)		TNNWGA($\overline{A}_i$)	
	$S(\overline{A}_i)$	*Ranking*	$S(\overline{A}_i)$	*Ranking*
$\overline{A}_1$	$S(\overline{A}_1)$=-0.5319	3	$S(\overline{A}_1)$=0.7868	3
$\overline{A}_2$	$S(\overline{A}_2)$=-0.7282	5	$S(\overline{A}_2)$=0.5865	5
$\overline{A}_3$	$S(\overline{A}_3)$=-0.6003	4	$S(\overline{A}_3)$=0.6293	4
$\overline{A}_4$	$S(\overline{A}_4)$=0.0306	1	$S(\overline{A}_4)$=1.3995	1
$\overline{A}_5$	$S(\overline{A}_5)$=-0.2591	2	$S(\overline{A}_5)$=1.0469	2

Table 7. The Sensitivity Analysis (One-Level-Up)

$\overline{A}_i$	TNNWAA($\overline{A}_i$)		TNNWGA($\overline{A}_i$)	
	$S(\overline{A}_i)$	*Ranking*	$S(\overline{A}_i)$	*Ranking*
$\overline{A}_1$	$S(\overline{A}_1)$=-0.5038	3	$S(\overline{A}_1)$=0.9469	3
$\overline{A}_2$	$S(\overline{A}_2)$=-0.6961	5	$S(\overline{A}_2)$=0.7346	5
$\overline{A}_3$	$S(\overline{A}_3)$=-0.5683	4	$S(\overline{A}_3)$=0.7822	4
$\overline{A}_4$	$S(\overline{A}_4)$=0.0584	1	$S(\overline{A}_4)$=1.4442	1
$\overline{A}_5$	$S(\overline{A}_5)$=-0.2409	2	$S(\overline{A}_5)$=1.2123	2

C. Comparative Analysis

A comparative analysis was conducted between the proposed methodology and the approach developed by Reference [39] to highlight both methodological distinctions and practical implications for decision-making.

1) Methodology

The two approaches differ primarily in design: the proposed methodology uses Trapezoidal Neutrosophic Numbers for presenting the fuzzy information, applies both arithmetic and geometric aggregation approaches, and employs a score function for defuzzification and ranking. In contrast, Reference [39] adopts Neutrosophic Trapezoidal Fuzzy Numbers. Rely only on arithmetic aggregation and apply the centroid method for ranking.

Table 8. Comparative Analysis (Methodology)

Item	Proposed Methodology	Reference [39]
Theoretical Basis	Trapezoidal Neutrosophic Numbers	Neutrosophic Trapezoidal Fuzzy Numbers
Ranking Method	Score Function	Centroid Method
Aggregation Method	Arithmetic and Geometric	Arithmetic

2) Ranking of Alternatives

The proposed method yields consistent results across both aggregation methods. In contrast, the comparative approach yields different rankings when the arithmetic operator is used, with some alternatives changing positions. Notably, applying the geometric aggregation method within the comparative framework results in rankings consistent with those produced by the proposed methodology. These findings indicate that the proposed methodology offers greater stability and that the geometric operator provides more reliable results than the arithmetic operator in the method of Reference [39].

Table 9. Comparative Analysis (Ranking of the Alternatives)

$\overline{A}_i$	Proposed Methodology		Reference [39]	
	TNNWAA($\overline{A}_i$)	*TNNWGA($\overline{A}_i$)*	*TNNWAA($\overline{A}_i$)*	*TNNWGA($\overline{A}_i$)*
$\overline{A}_1$	3	3	3	3
$\overline{A}_2$	5	5	1	5
$\overline{A}_3$	4	4	2	4
$\overline{A}_4$	1	1	4	1
$\overline{A}_5$	2	2	5	2

5. CONCLUSION

The results demonstrate the effectiveness of the proposed Multi-Criteria Decision Making (MCDM) method based on Trapezoidal Neutrosophic Numbers in selecting substrates for biogas recovery.

The final ranking identifies POMES as the most suitable substrate for biogas recovery, followed by other the alternatives in a stable and consistent order. Importantly, this result aligns with the initial assumption made by the decision-maker, further validating the accuracy and practical reliability of the proposed methodology.

The robustness of the proposed methodology was further confirmed through sensitivity and comparative analysis. The sensitivity analysis showed that the ranking of alternatives remains stable under both upward and downward perturbations of the input criteria, while the comparative analysis highlighted that the proposed method provides consistent rankings across aggregation methods and aligns closely with the geometric results of the initial studies.

Overall, the study confirms that our proposed methodology provides a useful tool for decision-making in biogas recovery selection. While this study primarily focuses on the technical methodology for substrate selection, we acknowledge the importance of practical factors, such as cost, availability, and environmental benefits, for industry adoption. Future research aims to incorporate these factors into consideration by evaluating the economic feasibility, geographical availability, and sustainability of the selected substrates.

Additionally, to foster broader application and adoption of this methodology, aligning future research with existing policy frameworks and industry standards can be critical. By integrating these standards, the proposed methodology could become part of best practices in biogas recovery, potentially influencing policy development and encouraging collaborative efforts across the renewable energy sector. This forward-looking perspective not only reinforces the utility of our findings but also invites industry stakeholders to engage in dialogue about opportunities for implementation.

ACKNOWLEDGEMENT

The authors wish to acknowledge the support of Universiti Teknologi MARA Sabah Branch, Kota Kinabalu Campus, Sabah, Malaysia.

REFERENCES

[1] L. A. Zadeh, “Fuzzy sets”, *Information and Control*, 1965, vol. 8, no. 3, pp. 338-353. doi: https://doi.org/10.1016/S0019-9958(65)90241-X.

[2] K. T. Atanassov, Intuitionistic Fuzzy Sets: Theory and Application. New York: Physica-Verlag, 1999, pp. 1-324. doi: https://doi.org/10.1007/978-3-7908-1870-3_1.

[3] F. Smarandache, “A unifying field in logics: Neutrosophic logic". In *Philosophy, American Research Press*, 1999, pp. 1-141.

[4] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, “Interval neutrosophic sets and logic: Theory and applications in computing”. Hexis, Phoenix, USA, 2005.

[5] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, “Single valued neutrosophic sets”, *Multisp Multistruct*, 2010, vol. 4, pp.410–413.

[6] S. Broumi, “Generalized Neutrosophic Soft Set”, International Journal of Computer Science, Engineering and Information Technology (IJCSEIT), 2013, vol. 3, no. 2, pp. 17 -30. doi: https://doi.org/10.5121/ijcseit.2013.3202.

[7] S. Broumi, S., Smarandache, and M. Dhar, “Rough neutrosophic sets”, *Infinite Study*, 2014, vol. 32, pp. 493-502. Doi: https://doi.org/10.5281/zenodo.30310.

[8] J. Ye, “A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets”, *J. Intell. Fuzzy Syst.*, 2014, vol. 26, no. 5, pp. 2459–2466. doi: https://doi.org/10.3233/IFS-130916.

[9] I. Deli, M. Ali, and F. Smarandache, “Bipolar neutrosophic sets and their application based on multi-criteria decision making problems”, In *2015 International conference on advanced mechatronic systems (ICAMechS*), *IEEE*, August 2015, pp. 249-254. doi: https://doi.org/10.1109/ICAMechS.2015.7287068.

[10] P. Ji, H. Y. Zhang, and J. Q. Wang, “A projection-based TODIM method under multi-valued neutrosophic environments and its application in personnel selection”, *Neural Comput. & Applic.*, 2018, vol. 29, pp. 221–234. doi: https://doi.org/10.1007/s00521-016-2436-z.

[11] J. J. Peng, J. Wang, and X. H. Wu, “An extension of the ELECTRE approach with multi-valued neutrosophic information”, *Neural Comput. & Applic.*, 2017, vol. 28, pp. 1011-1022. doi: https://doi.org/10.1007/s00521-016-2411-8.

[12] J. Ye, “Trapezoidal neutrosophic set and its application to multiple attribute decision making”, *Neural Comput. Appl.*, 2014, vol. 26, no. 5, pp. 1157–1166. doi: https://doi.org/10.1007/s00521-014-1787-6.

[13] I. Deli, and Y. Subas, Y, “Single valued neutrosophic numbers and their applications to multicriteria decision making problem”, *Neutrosophic Sets Systems*, 2014, vol. 2, no. 1, pp. 1-13.

[14] P. Biswas, S. Pramanik, and B.C. Giri, “Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making”, *Neutrosophic Sets and Systems*, 2016, vol. 12, pp. 127-138.

[15] I. Deli and Y. Subas, “A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems”, *Int. J. Mach. Learn. Cybern*, 2017, vol. 8, no. 4, pp. 1309–1322. doi: https://doi.org/10.1007/s13042-016-0505-3.

[16] S. I. A. Aal, M. M. Abd Ellatif, and M. M. Hassan, “Two ranking methods of single valued triangular neutrosophic numbers to rank and evaluate information systems quality”, *Infinite Study*, 2018.

[17] I. Deli, “Operators on single valued trapezoidal neutrosophic numbers and SVTN-group decision making”, *Neutrosophic Sets and Systems*, 2018, vol. 22, pp. 131-150.

[18] P. Liu, and X. Zhang, “Some Maclaurin symmetric mean operators for single-valued trapezoidal neutrosophic numbers and their applications to group decision making”, *International Journal of Fuzzy Systems*, 2018, vol. 20, pp. 45-61. doi: https://doi.org/10.1007/s40815-017-0335-9.

[19] S. Broumi, A. Bakali, M. Talea, F. Smarandache, and L. Vladareanu, “Computation of shortest path problem in a network with SV-trapezoidal neutrosophic numbers”, In 2016 *International Conference on Advanced Mechatronic Systems (ICAMechS)*, IEEE, November 2016, pp. 417-422. doi: https://doi.org/10.1109/ICAMechS.2016.7813484.

[20] I. Deli, “A novel defuzzification method of SV-trapezoidal neutrosophic numbers and multi-attribute decision making: A comparative analysis”, *Soft Computing*, 2019, vol. 23, no. 23, pp. 12529-12545. doi: https://doi.org/10.1007/s00500-019-03803-z.

[21] I. Deli, Theory of Single-Valued Trapezoidal Neutrosophic Numbers and their Applications to Multi Robot Systems. Toward Humanoid Robots: The Role of Fuzzy Sets. *A Handbook on Theory and Applications*, 2021, pp. 255-276. doi: https://doi.org/10.1007/978-3-030-67163-1_10 .

[22] R. K. Saini, A. Sangal, and M. Manisha, “Application of single valued trapezoidal neutrosophic numbers in transportation problem”, *Neutrosophic sets and systems*, 2020, vol. 35, no. 33, pp. 563-583. doi: https://doi.org/10.5281/zenodo.3965004.

[23] J. Ye, “Some weighted aggregation operators of trapezoidal neutrosophic numbers and their multiple attribute decision making method”, *Informatica*, 2017, vol. 28, no. 2, pp. 387-402.

[24] M. Abdel-Basset, M. Mohamed, and A. K. Sangaiah, “Neutrosophic AHP-Delphi Group decision making model based on trapezoidal neutrosophic numbers”, *Journal of Ambient Intelligence and Humanized Computing*, 2018, vol. 9, pp. 1427-1443. doi: https://doi.org/10.1007/s12652-017-0548-7.

[25] P. Biswas, S. Pramanik, and B.C. Giri, “TOPSIS strategy for multi-attribute decision making with trapezoidal neutrosophic numbers”, *Neutrosophic Sets and Systems*, 2018, vol. 19, pp. 29-39. doi: https://doi.org/10.5281/zenodo.1235335.

[26] S. Pramanik, and R. Mallick, “VIKOR based MAGDM strategy with trapezoidal neutrosophic numbers”, *Neutrosophic Sets and Systems*, 2018, vol. 22, pp. 118-129. doi: https://doi.org/10.5281/zenodo.2160840.

[27] D. Djatkov, E. Mathias and M. Milan, "Method for assessing and improving the efficiency of agricultural biogas plants based on fuzzy logic and expert systems", *Applied energy*, 2014, vol. 134, pp.163-175. doi: https://doi.org/10.1016/j.apenergy.2014.08.021.

[28] A. Ikpe, A. I. Ndon, and P. Etim, "Fuzzy modelling and optimization of anaerobic co-digestion process parameters for effective biogas yield from bio-wastes" *The International Journal of Energy and Engineering Sciences*, 2020, vol. 5, no.2, pp. 43-61.

[29] V. Başhan, and A. Y. Çetinkaya. "Application fuzzy DEMATEL methodology to investigate some technical parameters of biochemical methane potential (BMP) test produced vegetable waste anaerobic biogas”, *Environmental Monitoring and Assessment*, 2022, vol. 194, no. 661. doi: https://doi.org/10.1007/s10661-022-10330-2.

[30] A. Rahimieh, M. Mehriar, S. M. Zamir, and M. Nosrati, "Fuzzy-decision tree modeling for H2S production management in an industrial-scale anaerobic digestion process”, *Biochemical Engineering Journal*, 2024, vol. 208, no. 109380. doi: https://doi.org/10.1016/j.bej.2024.109380.

[31] O. Olatunji, et al., "Evolutionary optimization of biogas production from food, fruit, and vegetable (FFV) waste", *Biomass Conversion and Biorefinery*, 2024, vol. 14, pp. 12113-12125. doi: https://doi.org/10.1007/s13399-023-04506-0.

[32] R. Selaman, D. Indim, R. Rasmidi, T. Mohamed, and A. Lepit, "Biogas recovery from different composition of food waste using anaerobic digestion technique", In *APS Proceeding*, 2023, vol. 8, pp. 37-42.

[33] R. Selaman and T. P. Utomo, "The characterisation of protein-rich food waste and pond sludge for biomethane recovery", *Borneo Akademika*, 2024, vol. 8, no. 1, pp 1-7. doi: https://doi.org/10.24191/BA/v8i1/114022.

[34] A. Paranje, S. Saxena, and P. Jain, "A review in performance improvement of anaerobic digestion using co-digestion of food waste and sewage sludge", *Journal of Environment Management*, 2023, vol. 338, Art. no. 117733. doi: https://doi.org/10.1016/j.jenvman.2023.117733.

[35] S. Abd Hamid, N. Aini, and R. Selaman, "Anaerobic digestion of fruit wastes for biogas production", *IJARCCE*, 2019, vol 5, no.4, pp. 34-38.

[36] R. Kar, A. K. Shaw, and J. Mishra, "Trapezoidal fuzzy numbers (trfn) and its application in solving assignment problems by hungarian method: A new approach", *Fuzzy intelligent systems: methodologies, techniques, and applications*, 2021, pp. 315-333. doi: https://doi.org/10.1002/9781119763437.ch11.

[37] A. Abdillah, T. Hidaka, and T. Fujiwara, "Enhancing sludge stabilization and methane production in the anaerobic digestion of palm oil mill effluent using carbon conductive materials", *Journal of Materials Cycles and Waste Management*, 2025. doi: https://doi.org/10.1007/s10163-025-02317-1.

[38] S. Mohammad, S. Baidurah, T. Kobayashi, N. Ismail, and C. P. Leh, "Palm Oil Mill Effluent Treatment Processes—A Review", *Processes*, 2021, vol. 9, no.5, Art. no.739. doi: https://doi.org/10.3390/pr9050739.

[39] M. Suresh, K. Arun Prakash, and S. Vengataasalam, "Multi-criteria decision making based on ranking of neutrosophic trapezoidal fuzzy numbers", *Granular Computing*, 2021, vol. 6, pp. 943-952. doi: https://doi.org/10.1007/s41066-020-00240-4.