



Approximation Solutions of an Unsteady Fractional Viscoelastic Blood Flow Model in a Compliant Artery

Blessing Omojo Akogwu¹, Franklin Olusodayo Ogunfiditimi², and Ayinde Muhamed Abdullahi²

¹*Applied Mathematics and Simulation Advanced Research Centre, Sheda Science and Technology Complex, Sheda-Kwali, FCT, Nigeria.*

²*Department of Mathematics, Faculty of Science, University of Abuja, FCT, Nigeria*

KEYWORDS

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Viscoelasticity
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ABSTRACT

This paper develops and analyzes an unsteady fractional-order blood flow model in a compliant artery by incorporating Caputo fractional derivatives into the classical one-dimensional momentum equation. The fractional formulation accounts for viscoelastic and memory effects that integer-order models cannot adequately describe. Under physiological assumptions of constant blood density, constant viscosity, and a prescribed pressure gradient, approximate analytical solutions are derived using the Fractional Reduced Differential Transform Method (FRDTM). The Fractional Variational Iteration Method (FVIM) is used to validate the FRDTM solutions. Numerical results examine the influence of fractional order, arterial cross-sectional area, and pressure gradient on flow rate. The results indicate that flow rate increases with time for all parameter values and that lower fractional orders demonstrate stronger memory effects compared to higher orders. As the fractional order approaches unity, the model converges toward classical Newtonian behaviour. Close agreement between the results of FRDTM and FVIM confirms the reliability of the proposed method. The findings suggest that fractional modeling provides a more realistic representation of blood flow dynamics in compliant arteries.

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1. INTRODUCTION

Mathematical modelling of blood flow plays a fundamental role in the analysis and understanding of cardiovascular function and related pathological conditions.

Accurate blood flow models are essential for predicting hemodynamic behaviour, assessing arterial diseases such as stenosis and aneurysms, and supporting the design of biomedical devices. Classical blood flow theories often treat blood as a Newtonian fluid flowing through rigid or mildly deformable vessels, leading to simplified analytical formulations that have been widely used in cardiovascular research [1, 2]. While such models provide useful insights under certain conditions, they may fail to capture important physiological features associated with arterial compliance, pulsatility, and time-dependent behaviour of blood.

One-dimensional blood flow models derived from the Navier–Stokes equations have been extensively studied due to their balance between physical realism and mathematical tractability [3–5]. These models typically describe the relationship between volumetric flow rate, pressure gradient, and arterial geometry under assumptions such as incompressibility, axisymmetric flow, and constant viscosity.

A notable contribution in this area is the work of [6], who formulated a reduced blood flow model accounting for arterial compliance and pressure variation. Despite their usefulness, most existing models remain integer-order formulations, implicitly assuming that the fluid responds instantaneously to applied forces.

In the context of hemodynamic, fractional-order models have been introduced to better describe the non-Newtonian

*Corresponding author:

E-mail address: Blessing Omojo Akogwu <lucyblex1@gmail.com>.

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nature of blood and the viscoelastic behaviour of arterial walls [15–17]. These studies demonstrate that fractional derivatives significantly influence the temporal evolution of flow variables and can recover classical behaviour as the fractional order approaches unity. Nevertheless, many fractional blood flow models rely heavily on numerical discretization techniques or computationally expensive algorithms, which may limit analytical insight into the solution structure.

To address these challenges, several semi-analytical methods have been developed for solving fractional differential equations, including the Adomian Decomposition Method, Homotopy Perturbation Method, and Variational Iteration Method [18–20]. Among these techniques, the Fractional Reduced Differential Transform Method (FRDTM) has attracted attention due to its simplicity, fast convergence, and ability to generate accurate series solutions without linearization or small-parameter assumptions [21, 22]. Although FRDTM has been successfully applied to various fractional models in fluid mechanics and transport phenomena, its application to fractional blood flow models remains largely unexplored.

Motivated by these observations, the present study develops an unsteady fractional-order blood flow model in a compliant artery by extending the classical formulation of [22] through the introduction of Caputo fractional derivatives. The governing equation incorporates viscoelastic memory effects while maintaining physiologically meaningful assumptions. The resulting fractional model is solved using the Fractional Reduced Differential Transform Method, and the accuracy of the obtained solutions is validated through comparison with the Fractional Variational Iteration Method (FVIM). To the best of the authors' knowledge, this work represents one of the first attempts to apply FRDTM to an unsteady fractional blood flow model in a compliant arterial system.

The effects of key parameters, including fractional order, arterial cross-sectional area, and pressure gradient, on the blood flow rate are investigated in detail. The results provide insight into the role of fractional order in governing flow dynamics and demonstrate the transition from viscoelastic to classical Newtonian behaviour. The findings highlight the potential of fractional modelling combined with efficient semi-analytical techniques for advancing the mathematical analysis of cardiovascular flow systems.

2. MATHEMATICAL FORMULATION

2.1 Blood Flow Model Formulation

The present blood flow model is adopted from the work of [22]. In their formulation, the blood vessel is idealized as a straight, compliant cylindrical artery of length L , aligned along the axial coordinate z . A schematic representation of the arterial geometry is shown in Figure 1, illustrating the inlet, outlet, and intermediate (proximal, middle, and distal) sections of the vessel.

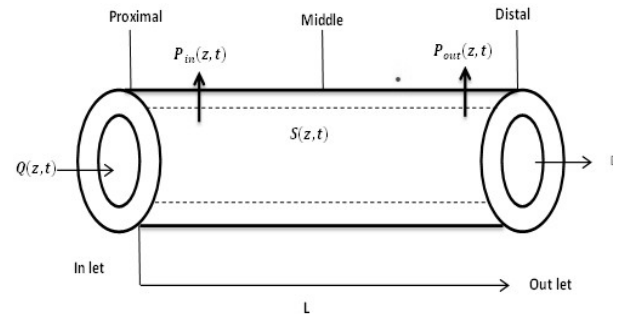


Fig. 1. Schematic Representation of the Arterial Geometry

The local cross-sectional area of the artery is denoted by $S(z, t)$, while $Q(z, t)$ and $P(z, t)$ represent the volumetric flow rate and blood pressure, respectively. Blood is treated as a Newtonian, incompressible fluid, and its motion is governed by the Navier–Stokes equations formulated in cylindrical coordinates. To simplify the governing equations, the following flow assumptions are made:

- The flow is axisymmetric.
- There is no tangential velocity component.
- Velocity components in the radial and circumferential directions are negligible.

Under these assumptions, the governing equations reduce to a one-dimensional blood flow model suitable for cardiovascular analysis.

It is important to note that the artery is described as compliant based on the underlying formulation, which incorporates the interaction between pressure and cross-sectional area. However, for analytical simplicity, the cross-sectional area is assumed to remain locally constant within a small arterial segment. As a result, arterial compliance is represented implicitly rather than through explicit wall elasticity parameters such as Young's modulus or wall thickness. This simplification allows the focus to remain on the influence of fractional-order dynamics on blood flow behaviour.

2.2 Governing Equations

Following [22], the one-dimensional momentum equation governing unsteady blood flow in a compliant artery is expressed as:

$$\frac{\partial Q}{\partial t} - \frac{3Q}{S} \frac{\partial Q}{\partial z} - \frac{2Q}{Q} \frac{\partial S}{\partial z} + \frac{4\pi\mu}{S} Q + \frac{S}{2\rho} \frac{\partial P}{\partial z} = 0 \quad (1)$$

The continuity (mass conservation) equation is given by:

$$\frac{\partial Q}{\partial z} + \frac{\partial S}{\partial t} = 0 \quad \text{or} \quad \frac{\partial Q}{\partial z} = -\frac{\partial S}{\partial t} \quad (2)$$

Substituting (2) into the momentum equation (2) yields the master cardiovascular system equation as;

$$\frac{\partial Q}{\partial t} + \frac{3Q}{S} \frac{\partial S}{\partial t} + \frac{2Q}{Q} \frac{\partial S}{\partial z} + \frac{4\pi\mu}{S} Q + \frac{S}{2\rho} \frac{\partial P}{\partial z} = 0 \quad (3)$$

This equation represents the general unsteady blood flow model accounting for arterial compliance and pressure variation.

The spatial variable z is defined along the axial direction of the artery and is restricted to a finite interval corresponding to the physical length of the vessel segment under consideration. Since the model is one-dimensional, only axial

variations are taken into account, while radial and circumferential variations are neglected under the assumption of axisymmetric flow.

2.3 Model Assumptions

To obtain an analytically tractable form of the governing equation, the following simplifying assumptions are adopted in accordance with [22].

1. The cross-sectional area is assumed to be constant in time and uniform over a small segment of the vessel,

$$\frac{\partial S}{\partial t} = 0, \frac{\partial S}{\partial z} = 0 \quad (4)$$

2. The axial pressure gradient $\frac{\partial P}{\partial z}$ is taken as a constant.
3. The blood density ρ and viscosity μ are assumed to remain constant throughout the flow domain.

Although blood exhibits non-Newtonian characteristics such as shear-thinning behaviour, it can be reasonably approximated as a Newtonian fluid with constant viscosity in large arteries where shear rates are sufficiently high. Furthermore, blood density is assumed constant due to its incompressible nature under normal physiological conditions. These assumptions are widely adopted in one-dimensional hemodynamic models to ensure mathematical tractability. In the present study, the introduction of the fractional-order derivative compensates for these simplifications by incorporating viscoelastic and memory effects, thereby enhancing the physical realism of the model without introducing excessive nonlinear complexity.

Applying these assumptions to (3), the governing equation simplifies to :

$$\frac{\partial Q}{\partial t} + \frac{4\pi\mu}{S} Q + \frac{S}{2\rho} \frac{\partial P}{\partial z} = 0 \quad (5)$$

where $z \in [0,2]$ is the spatial variable, S denotes the cross-sectional area of the artery, and $\frac{\partial P}{\partial z} = \Delta p$ represents the constant pressure gradient.

2.4 Unsteady Fractional Viscoelastic Blood Flow Model

The classical blood flow model described by (2) assumes an integer-order time derivative and therefore neglects memory and viscoelastic effects inherent in real blood flow. To overcome this limitation, the present study extends the model of [22] by incorporating fractional-order derivatives in the Caputo sense.

In this extension, the first-order time derivative is replaced by a fractional derivative of order $0 < \alpha < 1$, allowing the model to capture hereditary and viscoelastic properties of blood flow. The resulting unsteady fractional blood flow model is given by

$$\frac{\partial^\alpha Q}{\partial t^\alpha} = -\frac{4\pi\mu}{S} Q - \frac{S}{2\rho} \Delta p, \quad 0 < \alpha < 1, \quad z > 0 \text{ and } t \in [0,1] \quad (6)$$

The introduction of the fractional-order derivative enables the model to capture memory effects, which reflect the dependence of the current flow state on its past history. Physiologically, this corresponds to the delayed response of blood and arterial walls, where deformation and relaxation do not occur instantaneously. In contrast to classical integer-order models, the fractional formulation accounts for gradual recovery of the arterial wall between successive pulses,

thereby providing a more realistic representation of viscoelastic behaviour in cardiovascular flow systems.

This fractional formulation represents a direct generalization of the integer-order model and serves as the primary governing equation for the present study.

Subject to the initial condition

$$Q(z, 0) = 16.67 \text{ cm}^2/\text{s} \quad (7)$$

where α is the fractional order and $\frac{\partial^\alpha Q}{\partial t^\alpha}$ is the Caputo derivative of order α .

2.5 Caputo Fractional Derivative

The Caputo fractional derivative of order α with respect to time for the blood flow rate

$Q(t)$ is defined as:

$$D_t^\alpha Q(t) = \frac{1}{\Gamma(1-\alpha)} \int_a^t \frac{\partial Q(\tau)}{(t-\tau)^\alpha} d\tau, \quad 0 < \alpha \leq 1 \quad (8)$$

where $\Gamma(\cdot)$ denotes the Gamma function.

2.6 Physiological Parameters

All physical parameters are adopted from [22] and summarized in Table 1.

Table 1. Physiological Parameters used in the Model.

Variables and parameters	Descriptions	Values
Q	Blood Flow rate	16.67 cm ² /s
S	Cross Sectional Area	[0.5, 1.0, 1.5, 2.0]cm ²
Δp	Pressure Gradient	[-40, -60, -80, -100]mmH
π	Pi	3.14159
M	Mu	0.035cm ² /s
P	Rho	1.050 g/cm ³

The initial blood flow rate is fixed at $Q = 16.67 \text{ cm}^2/\text{s}$, consistent with the reference model [22], and represents a physiologically realistic baseline condition. Maintaining a constant initial flow rate across all simulations ensures consistency and allows for a clear assessment of the effects of fractional order, pressure gradient, and cross-sectional area on the flow dynamics.

3. METHOD OF SOLUTION

3.1 Fractional Reduced Differential Transform Method (FRDTM)

In this section, we apply the Fractional Reduced Differential Transform Method for solving the fractional blood flow model in (5). The Fractional Reduced Differential Transform Method (FRDTM) is a semi-analytical technique employed to solve fractional differential equations. It extends the traditional Reduced Differential Transform Method to handle fractional derivatives, offering solutions in the form of convergent series [20, 21].

3.2 Basic Concept of Fractional Reduced Differential Transform Method (FRDTM)

Let $f(x,t)$ be a function of two variables expressed as product of functions as;

$$f(x, t) = u(x)v(t) \quad (9)$$

Based on theorem 2 of fractional differential transform method [23], we have

$$f(x, t) = \sum_{i=0}^{\infty} u(i)x^i \times \sum_{j=0}^{\infty} v(j)t^j = \sum_{k=0}^{\infty} F_k(x)t^k \quad (10)$$

where F_k is the t -dimensional spectrum function of $f(x, t)$

Definition 1: According to [24], If the function $f(x, t)$ is analytical and k -times continuously differentiable with α^{th} derivatives with respect to the time t and space x in the domain of interest, then let the Fractional Reduced Differential Transform (FRDT) of $f(x, t)$ be given as

$$\text{FRDT}[f(x, t)] = F_k(x) = \frac{1}{\Gamma(k\alpha+1)} \left[(D_{a,t}^{\alpha})^k (f(x, t)) \right]_{t=t_0} = \frac{1}{\Gamma(k\alpha+1)} \left[\frac{\partial^{k\alpha}}{\partial t^{k\alpha}} f(x, t) \right]_{t=t_0}, k = 0, 1, 2, \dots \quad (11)$$

where $\alpha > 0$ the time fractional order derivative, t -dimensional spectrum function $F_k(x)$ is the transformation function and $f(x, t)$ is the original function.

Definition 2: The differential inverse fractional reduced transform of $F_k(x)$ denoted by $f(x, t)$ is given by

$$\text{FRDT}^{-1}(F_k(x)) = f(x, t) = \sum_{k=0}^{\infty} F_k(x)(t - t_0)^{k\alpha} \quad (12)$$

Combining (11) and (12), we have

$$f(x, t) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k\alpha+1)} \left[\frac{\partial^{k\alpha}}{\partial t^{k\alpha}} f(x, t) \right]_{t=t_0} (t - t_0)^{k\alpha} \quad (13)$$

Equation (13) shows clearly that the concept of the Fractional Reduced Differential Transform is derived from the power series expansion.

3.3 Theorems of Fractional Reduced Differential Transform Method (FRDTM)

Let $f(x, t)$, $u(x, t)$ and $v(x, t)$ be analytical and k -times continuously differentiable functions with respect to the space variable x and time variable t , then the following Theorems holds [25]

Theorem 1. If $f(x, t) = u(x, t)$, then $F_k(x, t) = U_k(x, t)$

Theorem 2. If $u(x, t) = u(x, t) \pm v(x, t)$, then $F_k(x, t) = U_k(x, t) \pm V_k(x, t)$

$$F_k(x, t) = U_k(x, t) \pm V_k(x, t).$$

Theorem 3. If $f(x, t) = cu(x, t)$, then $F_k(x, t) = cU_k(x, t)$, where c is a constant.

Theorem 4. If $f(x, t) = \frac{\partial^n}{\partial x^n} u(x, t)$, then $F_k(x, t) = \frac{\partial^n}{\partial x^n} U_k(x, t)$

Theorem 5: if $f(x, t) = \frac{\partial^{\alpha n}}{\partial t^{\alpha n}} u(x, t)$, then $F_k(x, t) = \frac{\Gamma(\alpha(k+n)+1)}{\Gamma(k\alpha+1)} U_{k+n}(x)$, $n = 1, 2, 3, \dots$

Theorem 6. If $f(x, t) = u(x, t)v(x, t)$ then $F_k = \sum_{i=0}^k U_i(x)V_{k-i}(x)$

3.4 Implementation of the Fractional Reduced Differential Transform Method

Here we apply the Fractional Reduced Differential Transform (FRDT) to both sides of (7) and using Theorem 1 and 5, (5) is transformed to

$$\frac{\Gamma(\alpha(k+1)+1)}{\Gamma(k\alpha+1)} q_{k+1}(z) = -\frac{4\pi\mu}{S} q_k(z) - \frac{S}{2\rho} \Delta p \quad (14)$$

Or equivalently,

$$q_{k+1}(z) = \frac{\Gamma(k\alpha+1)}{\Gamma(\alpha(k+1)+1)} \left(-\frac{4\pi\mu}{S} q_k(z) - \frac{S}{2\rho} \Delta p \right), k = 0, 1, 2, 3, \dots \quad (15)$$

The initial condition in (6) is transformed to;

$$q_0 = 16.67 \text{ cm}^2/\text{s} \quad (16)$$

Using (16), $q_{k+1}(z)$ is computed as;

for $k = 0$,

$$q_{0+1}(z) = \frac{\Gamma(0+1)}{\Gamma(\alpha(0+1)+1)} \left(-\frac{4\pi\mu}{S} q_0(z) - \frac{S}{2\rho} \Delta p \right)$$

$$q_{0+1}(z) = \frac{\Gamma(0+1)}{\Gamma(\alpha(0+1)+1)} \left[-\frac{4 \times 3.141593 \times 0.035}{0.5} \times 16.6667 - \frac{0.5}{2 \times 1.050} \times (-80) \right]$$

$$q_1(z) = \frac{\Gamma(1)}{\Gamma(\alpha+1)} [-14.66079504 + 19.04761905]$$

$$q_1(z) = \frac{1}{\Gamma(\alpha+1)} (4.3868401) = \frac{4.38682401}{\Gamma(\alpha+1)}$$

for $k = 1$

$$q_{1+1}(z) = \frac{\Gamma(\alpha+1)}{\Gamma(\alpha(1+1)+1)} \left(-\frac{4\pi\mu}{S} q_1(z) - \frac{S}{2\rho} \Delta p \right)$$

$$q_2(z) = \frac{\Gamma(\alpha+1)}{\Gamma(2\alpha+1)} \left(-14.66079504 \times \left(\frac{4.3868401}{\Gamma(\alpha+1)} \right) - \frac{0.5}{2 \times 1.050} \times (-80) \right)$$

$$= -\frac{3.858852369}{\Gamma(2\alpha+1)} + \frac{19.04761906\Gamma(\alpha+1)}{\Gamma(2\alpha+1)}$$

for $k = 2$

$$q_{2+1}(z) = \frac{\Gamma(2\alpha+1)}{\Gamma(\alpha(2+1)+1)} \left(-\frac{4\pi\mu}{S} q_2(z) - \frac{S}{2\rho} \Delta p \right)$$

$$q_3(z) = \frac{\Gamma(2\alpha+1)}{\Gamma(3\alpha+1)} \left(-14.66079504 \times \left(-\frac{3.858852369}{\Gamma(2\alpha+1)} + \frac{19.04761906\Gamma(\alpha+1)}{\Gamma(2\alpha+1)} \right) - \frac{0.5}{2 \times 1.050} \times (-80) \right)$$

$$= \frac{3.394424205}{\Gamma(3\alpha+1)} - \frac{279.2532389\Gamma(\alpha+1)}{\Gamma(3\alpha+1)} + \frac{19.04761906\Gamma(\alpha+1)}{\Gamma(3\alpha+1)}$$

:

Using the inverse transform function in (12), the approximate solutions $Q(z, t)$ is obtained as

$$Q(z, t) = \sum_{k=0}^{10} q_k(z) t^{\alpha k}, k = 0, 1, 2, 3, 4, \dots \text{ and } \alpha = 0.25, 0.50, 0.75 \text{ and } 1.0 \quad (17)$$

$$q_0(z) + q_1(z) t^{\alpha} + q_2(z) t^{2\alpha} + q_3(z) t^{3\alpha} + \dots$$

$$q(z, t) = 16.67 + \left(\frac{4.38682401}{\Gamma(\alpha + 1)} \right) t - \left(\frac{3.858852369}{\Gamma(2\alpha + 1)} - \frac{19.04761906\Gamma(\alpha + 1)}{\Gamma(2\alpha + 1)} \right) t^{\alpha} + \left(\frac{3.394424205}{\Gamma(3\alpha + 1)} - \frac{279.2532389\Gamma(\alpha + 1)}{\Gamma(3\alpha + 1)} + \frac{19.04761906\Gamma(\alpha + 1)}{\Gamma(3\alpha + 1)} \right) t^{2\alpha} + \dots$$

The FRDTM algorithm was implemented to compute the series coefficients $q_k(z)$ and recursively up to the 10th term iteration ($N = 10$). The resulting solutions were validated through graphical comparison and absolute error analysis against FVIM results.

3.5 Implementation of the Fractional Variational Iteration Method (FVIM) for Validation

To validate the FRDTM results, the same model was solved using the Fractional Variational Iteration Method (FVIM). The correction functional for equation (5) is defined as;

$$Q_{k+1}(z, t) = Q_k(z, t) + \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} \lambda(\tau) \left[D_{\tau}^{\alpha} Q_k(z, \tau) + \frac{4\pi\mu}{S} \times Q_k(z) - \frac{S}{2\rho} \times \Delta p \right] d\tau \quad (18)$$

where $\lambda(\tau) = -1$ is the Lagrange multiplier. Starting with the initial approximation $Q(z, 0) = Q_0(z, t)$, the successive corrections $Q_{k+1}(z, t)$ were computed up to the 15th iteration term. We define the constants in (18) for ease of computation as;

$$a = \frac{4\pi\mu}{S}(z), \quad b = \frac{S}{2\rho} \times \Delta p \quad (19)$$

Substituting (19) in (18), the governing fractional differential equation becomes

$D_{\tau}^{\alpha} Q(z, t) + aQ(z, t) = -b$, so that (18) becomes

$$Q_{k+1}(z, t) = Q_k(z, t) - \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} [D_{\tau}^{\alpha} Q_k(z, \tau) + aQ_k(z, \tau) + b] d\tau \quad (20)$$

for $k = 0$

$$Q_1(z, t) = Q_0(z, t) - \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} [D_{\tau}^{\alpha} Q_0(z, \tau) + aQ_0(z, \tau) + b] d\tau \quad (21)$$

Because $Q_0(z, t)$ contains no explicit time dependence, its Caputo derivative vanishes, so that (21) reduces to

$$Q_1(z, t) = Q_0(z, t) - \frac{[aQ_0(z, t) + b]}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} d\tau \quad (22)$$

Evaluating the integral, we obtain

$$\int_0^t (t - \tau)^{\alpha-1} d\tau = \frac{t^{\alpha}}{\Gamma(\alpha+1)} \quad (23)$$

So that

$$Q_1(z, t) = Q_0(z, t) - \frac{[aQ_0(z, t) + b]}{\Gamma(\alpha+1)} = Q_0 - \frac{1}{\Gamma(\alpha+1)} \left(Q_0 \frac{4\pi\mu}{S} + \frac{S}{2\rho} \times \Delta p \right) \quad (24)$$

for $k = 1$

$$Q_2(z, t) = Q_1(z, t) - \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} [D_{\tau}^{\alpha} Q_1(z, \tau) + aQ_1(z, \tau) + b] d\tau \quad (25)$$

Expanding (25),

$$Q_1(z, \tau) = Q_0(z, \tau) - \frac{[aQ_0(z, \tau) + b]\tau^{\alpha}}{\Gamma(\alpha+1)} \quad (26)$$

Substitute (26) in (25) and integrate term by term, we have

$$Q_2(z, t) = Q_0(z, t) - \frac{[aQ_0(z, t) + b]t^{\alpha}}{\Gamma(\alpha+1)} + \frac{a[aQ_0(z, t) + b]t^{2\alpha}}{\Gamma(2\alpha+1)} \quad (27)$$

:

The general solution is given as

$$Q_{n+1}(z, t) = Q_0(z, t) \sum_{k=0}^{\infty} \frac{(-at^{\alpha})^k}{\Gamma(\alpha+1)} - \frac{b}{a} \left[1 - \sum_{k=0}^{\infty} \frac{(-at^{\alpha})^k}{\Gamma(\alpha+1)} \right] \quad (28)$$

In terms of computational performance, the Fractional Reduced Differential Transform Method (FRDTM) demonstrates a simpler implementation and faster convergence compared to the Fractional Variational Iteration Method (FVIM). The FRDTM relies on a direct recursive formulation to compute series coefficients, avoiding repeated integrations. In contrast, FVIM involves iterative correction functionals and integral evaluations, which increase computational complexity. In the present study, accurate results were obtained using fewer iterations in FRDTM ($N = 10$) compared to FVIM ($N = 15$), highlighting the efficiency of the method.

4. RESULTS AND DISCUSSION

In this section, we analyze the approximate solutions obtained using the Fractional Reduced Differential Transform Method (FRDTM) and the Fractional Variational Iteration Method (FVIM) for equation 5, focusing on blood flow under various scenarios.

- Case (i): The cross-sectional area is varied while the fractional order and the pressure gradient remain fixed.
- Case (ii): The pressure gradient is varied, while the fractional order and the cross-sectional area are fixed.
- Case (iii): The fractional order is varied, while the cross-sectional area and the pressure gradient are maintained as constants over the time interval.

Additionally, the influence of the fractional order on the blood flow model is examined, and all computations are performed using Maple 2021, with the results presented in tabular and graphical form.

Table 2. The Numerical Results of the Blood Flow Rate $Q(z, t)$ computed using FRDTM and FVIM for varying Cross-sectional Area $S = (0.5, 1.0, 1.5 \text{ and } 2.0) \text{ cm}^2$, with a fixed Fractional-Order $\alpha = 1.0$ and Pressure Gradient $\Delta p = -80 \text{ mmHg}$, over the interval $t \in (0, 1]$

t	FRDTM $S=0.5$	FVIM $S=0.5$	FRDTM $S=1.0$	FVIM $S=1.0$	FRDTM $S=1.5$	FVIM $S=1.5$	FRDTM $S=2.0$	FVIM $S=2.0$
0	16.6700000	16.6700000	16.6700000	16.6700000	16.6700000	16.6700000	16.6700000	16.6700000
0.1	17.0866417	17.0866416	19.6765097	19.6765097	21.8164246	21.8164247	23.8400634	23.8400634
0.2	17.4712214	17.4712214	22.5568100	22.5568100	26.8173438	26.8173437	30.8573982	30.8573982
0.3	17.8234169	17.8234169	25.3131736	25.3131735	31.6737573	31.6737571	37.7220982	37.7220983
0.4	18.1459553	18.1459553	27.9509333	27.9509332	36.3898407	36.3898406	44.4374834	44.4374834
0.5	18.4413338	18.4413337	30.4751926	30.4751925	40.9696489	40.9696489	51.0068016	51.0068017
0.6	18.7118394	18.7118393	32.8908353	32.8908352	45.4171198	45.4171198	57.4332298	57.4332298
0.7	18.9595666	18.9595665	35.2025350	35.2025350	49.7360773	49.7360774	63.7198761	63.7198760
0.8	19.1864335	19.1864334	37.4147644	37.4147643	53.9302350	53.9302351	69.8697809	69.8697809
0.9	19.3941967	19.3941967	39.5318035	39.5318034	58.0031991	58.0031991	75.8859184	75.8859180
1.0	19.5844648	19.5844648	41.5577484	41.5577483	61.9584715	61.9584719	81.7711983	81.7711983

Table 2 shows the results obtained by the FRDTM and the FVIM solutions for fractional-order $\alpha = 1.0$, with fixed Pressure gradient $\Delta p = -80 \text{ mmHg}$ and different cross-sectional area $S = (0.5, 1.0, 1.5 \text{ and } 2.0) \text{ cm}^2$, over $t \in (0, 1]$. It is observed that the blood flow rate Q increases monotonically with time t for all cross-sectional areas S . This pattern reflects the progressive response of arterial flow under a fixed pressure gradient of $\Delta p = -80 \text{ mmHg}$. As the cross-sectional area S increases from 0.5 cm^2 to 2.0 cm^2 , the corresponding flow rate Q also increases from $17.0866416 \text{ cm}^2/\text{s}$ to $23.840063 \text{ cm}^2/\text{s}$ at $t = 0.1 \text{ s}$ and from $19.5844648 \text{ cm}^2/\text{s}$ to $81.7711983 \text{ cm}^2/\text{s}$ at $t = 1.0 \text{ s}$, demonstrating that the flow is proportional to vessel area under a constant pressure gradient, confirming that Poiseuille's law is preserved.

$\alpha = 1.0$, with fixed Pressure Gradient $\Delta p = -80 \text{ mmHg}$ and varying Cross-sectional Area $S = (0.5, 1.0, 1.5 \text{ and } 2.0) \text{ cm}^2$, over the interval $t \in (0, 1]$

Figure 2 shows that at $\alpha = 1.0$, the fractional derivative reduces to the integer first-order derivative, indicating the absence of memory or viscoelastic effects. The system behaves as a Newtonian fluid, responding instantaneously to any change in the applied pressure gradient. Physically, this corresponds to normal, laminar blood flow, where viscosity is constant and flow adjusts proportionally to the vessel geometry and driving pressure. In addition, the results obtained from the FRDTM and FVIM solutions are nearly identical across all values of t and S . However, the FRDTM demonstrates a simpler computational structure and achieves faster convergence compared to the FVIM.

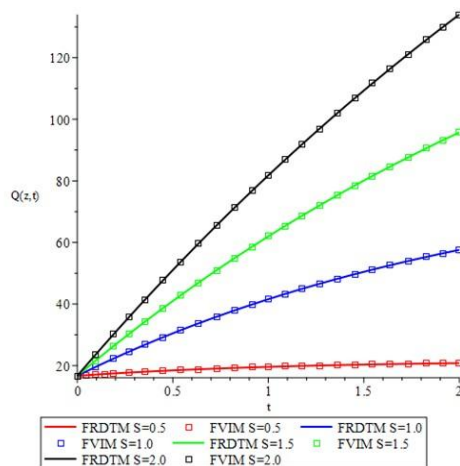


Fig. 2. 2D Plot Showing the Variation of Blood Flow Rate $Q(z, t)$ obtained by FRDTM and FVIM for Fractional-Order

Table 3. Numerical Results of Blood Flow Rate $Q(z, t)$ computed by FRDTM and FVIM for Fractional-Order $\alpha = 0.25$, with fixed Pressure Gradient $\Delta p = -80 \text{ mmHg}$ and varying Cross-sectional Area $S = (0.5, 1.0, 1.5 \text{ and } 2.0) \text{ cm}^2$, over the interval $t \in (0, 1]$

t	FRDTM $S=0.5$	FVIM $S=0.5$	FRDTM $S=1.0$	FVIM $S=1.0$	FRDTM $S=1.5$	FVIM $S=1.5$	FRDTM $S=2.0$	FVIM $S=2.0$
0	16.6700000	16.6700000	16.6700000	16.6700000	16.6700000	16.6700000	16.6700000	16.6700000
0.1	18.4606743	18.4606742	31.8621906	31.8621906	44.3738094	44.3738094	56.580194	56.5801942
0.2	18.66687952	18.6668795	34.0583338	34.0583338	48.72645481	48.72645481	63.1320713	63.1320712
0.3	18.78791791	18.7879179	35.4395291	35.4395291	51.51440231	51.51440231	67.3715652	67.3715652

t	FRDTM $S=0.5$	FVIM $S=0.5$	FRDTM $S=1.0$	FVIM $S=1.0$	FRDTM $S=1.5$	FVIM $S=1.5$	FRDTM $S=2.0$	FVIM $S=2.0$
0.4	18.87043126	18.87043127	36.46149170	36.4614917	53.60311999	53.60311999	70.5701188	70.5701188
0.5	18.92921352	18.92921352	37.27745669	37.2774567	55.28699518	55.28699517	73.1628315	73.1628313
0.6	18.97070000	18.97070002	37.95877704	37.9587771	56.70424555	56.70424554	75.3548781	75.3548781
0.7	18.99813898	18.99813898	38.54477584	38.5447758	57.93155215	57.9315521	77.2604880	77.2604879
0.8	19.01329886	19.01329888	39.05953622	39.0595362	59.01614393	59.01614394	78.9502264	78.9502264
0.9	19.01717018	19.01717019	39.51892380	39.5189238	59.9892891	59.98928911	80.4709338	80.4709338
1.0	19.01029774	19.01029774	39.93396658	39.9339666	60.87281854	60.87281856	81.8553899	81.8553899

Table 3 shows how the blood flow rate $Q(z, t)$ changes over time when the fractional order is $\alpha = 0.25$ and the pressure gradient is fixed at $\Delta p = -80\text{mmHg}$. The results are presented for four different cross-sectional areas $S = [0.5, 1.0, 1.5 \text{ and } 2.0]\text{cm}^2$. At this low fractional order $\alpha = 0.25$, the flow responds more slowly than in the classical (integer-order) case, showing a clear viscoelastic or memory-driven behaviour. Between $t = 0.1\text{s}$ and $t = 1.0\text{s}$, the discharge grows only slightly, by about 3% for the smallest area ($S = 0.5\text{cm}^2$), but the increase becomes more noticeable for larger areas, reaching roughly 25% for $S = 1.0\text{cm}^2$, 37% $S = 1.5\text{cm}^2$, and 45% for $S = 2.0\text{cm}^2$.

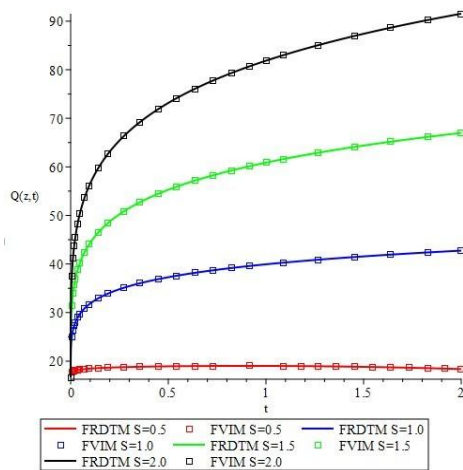


Fig. 3. 2D Plot Showing the Variation of Blood Flow Rate $Q(z, t)$ obtained by FRDTM and FVIM for Fractional

Order $\alpha = 0.25$, with fixed Pressure Gradient $\Delta p = -80\text{mmHg}$ and varying Cross-sectional Area $S = (0.5, 1.0, 1.5 \text{ and } 2.0)\text{cm}^2$, over the interval $t \in (0, 1]$.

As seen in Figure 3, the cross-sectional area $S = 0.5\text{cm}^2$ has the lowest flow behaviour, this is due to high resistance as a result of severe narrowing of the vessel. $S = 1.0\text{cm}^2$ has a moderate flow behavior (the flow slightly high) as a result of reduced stenosis, $S = 1.5\text{cm}^2$ has a higher flow due to less resistance to flow and $S = 2.0\text{cm}^2$ has the maximum flow due to minimal obstructions. Also, the flow rate rises steadily with time for all cross-sectional areas, meaning the vessel gradually adjusts to the applied pressure. Interestingly, the ratio Q/S remains nearly constant (around 40 near $t = 1.0\text{s}$), which suggests that the normal pressure–area relationship of laminar flow (similar to Poiseuille’s law) is maintained. The main effect of the fractional order $\alpha = 0.25$ is therefore not on the magnitude of flow but on how smoothly and slowly it evolves with time. Both FRDTM and FVIM give almost identical results, confirming that the numerical solutions.

Table 4. Numerical Results of Blood Flow Rate $Q(z, t)$ computed by FRDTM and FVIM for Fractional-Order $\alpha = 0.5$, with fixed Pressure Gradient $\Delta p = -80\text{mmHg}$ and varying Cross-sectional Area $S = (0.5, 1.0, 1.5 \text{ and } 2.0)\text{cm}^2$, over the interval $t \in (0, 1]\text{s}$.

t	FRDTM $S=0.5$	FVIM $S=0.5$	FRDTM $S=1.0$	FVIM $S=1.0$	FRDTM $S=1.5$	FVIM $S=1.5$	FRDTM $S=2.0$	FVIM $S=2.0$
0	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000
0.1	17.43072090	17.43072091	22.31016186	22.31016186	26.42282027	26.42282029	30.32770706	30.32770706
0.2	17.86655418	17.86655418	25.82650475	25.82650474	32.68567134	32.68567133	39.22964711	39.22964711
0.3	18.20136967	18.20136968	28.70767409	28.70767410	37.93236026	37.93236028	46.77140943	46.77140942
0.4	18.47514539	18.47514539	31.19947241	31.19947241	42.55967870	42.55967871	53.48938466	53.48938467
0.5	18.70631438	18.70631436	33.41280461	33.41280462	46.74414469	46.74414470	59.62023512	59.62023512
0.6	18.90552319	18.90552319	35.41117719	35.41117719	50.58579411	50.58579412	65.29719328	65.29719328
0.7	19.07968145	19.07968145	37.23579298	37.23579298	54.14902820	54.14902819	70.60555647	70.60555647
0.8	19.23360894	19.23360895	38.91555411	38.91555410	57.47876325	57.47876325	75.60453647	75.60453648
0.9	19.37083717	19.37083717	40.47184994	40.47184995	60.60810460	60.60810460	80.33760217	80.33760217
1.0	19.49404733	19.49404733	41.92114053	41.92114052	63.56245925	63.56245926	84.83800302	84.83800303

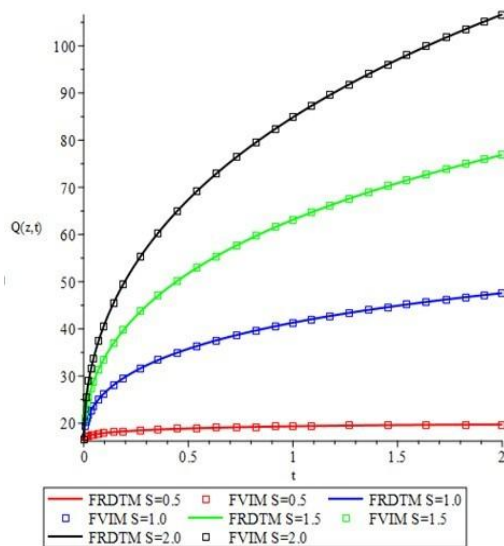


Fig. 4. 2D Plot Showing the Variation of Blood Flow Rate $Q(z, t)$ obtained by FRDTM and FVM for Fractional-Order $\alpha = 0.5$, with fixed Pressure Gradient $\Delta p = -80\text{mmHg}$ and varying Cross-sectional Area $S = (0.5, 1.0, 1.5 \text{ and } 2.0)\text{cm}^2$, over the interval $t \in (0, 1]$

Table 4 and Figure 4 present the blood flow rate $Q(z, t)$ for the fractional order $\alpha = 0.5$ at different cross-sectional areas $S = [0.5, 1.0, 1.5 \text{ and } 2.0]\text{cm}^2$, under a constant pressure gradient of $\Delta p = -80\text{mmHg}$. The results show that $Q(z, t)$ increases smoothly with time t across all vessel sizes, reflecting the gradual development of flow due to the fractional (viscoelastic) effects. Between $t = 0.1\text{s}$ and $t = 1.0\text{s}$, the flow rate rises by about 12% for $S = 0.5\text{cm}^2$, 47% for $S = 1.0\text{cm}^2$, 58% for $S = 1.5\text{cm}^2$, and 64% for $S = 2.0\text{cm}^2$. This demonstrates that larger vessel areas experience more pronounced increases in discharge over time. The nearly proportional growth of Q with S confirms the Poiseuille-like dependence of volumetric flow on cross-sectional area. Compared to the case of $\alpha = 0.25$, the flow evolution here is noticeably faster and less damped, indicating weaker memory effects. The system responds more promptly to the applied pressure gradient, showing a transition toward more Newtonian-like behaviour. The FRDTM and FVM solutions are virtually identical across all time levels and vessel sizes, confirming the accuracy and stability of both approaches.

Table 5. Numerical Results of Blood Flow rate $Q(z, t)$ computed by FRDTM and FVM for Fractional-Order $\alpha = 1.0$, with fixed Cross-sectional Area $S = 1.0\text{cm}^2$ and varying Pressure Gradients $\Delta p = [-40, -60, -80, \text{ and } -100]\text{mmHg}$ over interval $t \in (0, 1]$

t	FRDTM 40	FVM 40	FRDTM 60	FVM 60	FRDTM 80	FVM 80	FRDTM 100	FVM 100
0	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000
0.1	17.81302829	17.81302829	18.74476900	18.74476900	19.67650971	19.67650971	20.60825042	20.60825042
0.2	18.91003111	18.91003111	20.73342056	20.73342056	22.55681001	22.55681001	24.38019946	24.38019946
0.3	19.95983087	19.95983088	22.63650225	22.63650222	25.31317358	25.31317358	27.98984492	27.98984493
0.4	20.96445870	20.96445870	24.45769601	24.45769600	27.95093329	27.95093330	31.44417059	31.44417059
0.5	21.92585829	21.92585830	26.20052543	26.20052544	30.47519259	30.47519259	34.74985972	34.74985973
0.6	22.84588974	22.84588973	27.86836250	27.86836250	32.89083526	32.89083527	37.91330802	37.91330803
0.7	23.72633305	23.72633304	29.46443402	29.46443403	35.20253501	35.20253502	40.94063601	40.94063599
0.8	24.56889167	24.56889166	30.99182803	30.99182802	37.41476440	37.41476439	43.83770076	43.83770075
0.9	25.37519575	25.37519575	32.45349964	32.45349963	39.53180352	39.53180353	46.61010743	46.61010742
1.0	26.14680529	26.14680529	33.85227682	33.85227683	41.55774838	41.55774838	49.26321992	49.26321992

The negative sign of the pressure gradient indicates the direction of flow, with pressure decreasing along the axial direction of the artery. This is consistent with physiological blood flow, where fluid moves from regions of higher pressure to lower pressure. The selected range of pressure gradients (-40 to -100 mmHg) reflects realistic arterial pressure differences observed in the cardiovascular system.

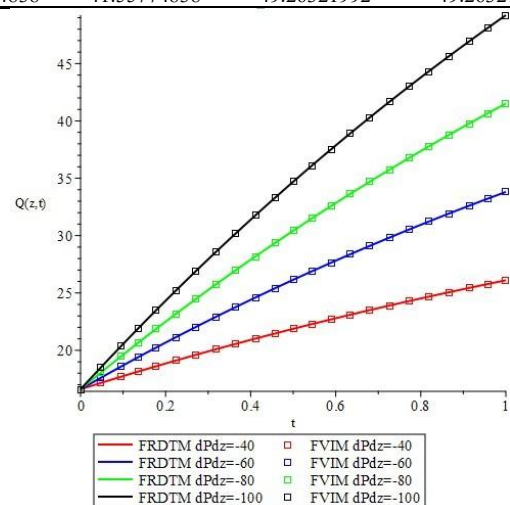


Fig. 5. 2D Plot Showing the Variation of Blood Flow Rate $Q(z, t)$ obtained by FRDTM and FVM for Fractional-

Order $\alpha = 1.0$, with fixed Cross-sectional Area $S = 1.0\text{cm}^2$ and varying Pressure gradients $\Delta p = [-40, -60, -80, \text{ and } -100]\text{mmHg}$, over the interval $t \in (0,1]$

Table 5 and Figure 5 presents the numerical results of the blood flow rate $Q(z,t)$ computed using the FRDTM and FVIM for the integer-order case $\alpha = 1.0$, at a fixed cross-sectional area $S = 1.0\text{cm}^2$ and pressure gradients $\Delta p = (-40, -60, -80, \text{ and } -100)\text{mmHg}$, over the interval $t \in (0,1]\text{s}$. It is evident that $Q(z,t)$ increases monotonically with time t for all applied pressure gradients. The flow development is rapid, indicating an instantaneous and memory-free response, characteristic of a classical Newtonian fluid. As the fractional derivative order reaches unity, the model simplifies to the standard integer-order differential equation, and thus no hereditary (viscoelastic) effects are

observed. Also, the magnitude of $Q(z,t)$ strongly depends on the applied pressure gradient: at $\Delta p = -40$, the flow rate increases from $16.67\text{cm}^2/\text{s}$ at $t = 0\text{s}$ to $26.14680529\text{cm}^2/\text{s}$ at $t = 1.0\text{s}$. At $\Delta p = -100\text{mmHg}$, it rises more sharply from $16.67\text{cm}^2/\text{s}$ to $49.26321992\text{cm}^2/\text{s}$, representing an approximately 338% increase over the time interval. This shows a clear direct proportionality between Q and the pressure gradient, fully consistent with Poiseuille's law, where $Q \propto \Delta p$. The time evolution of Q is smooth and linear-like, reflecting laminar flow behavior in a rigid, Newtonian system. Furthermore, the close numerical agreement between FRDTM and FVIM results (identical up to at least six decimal places) validates the consistency of FRDTM.

Table 6. Numerical Results of Blood Flow Rate $Q(z,t)$ computed by FRDTM and FVIM for Fractional-Order $\alpha = 0.25$, with fixed Cross-sectional Area $S = 1.0\text{cm}^2$ and varying Pressure Gradients $\Delta p = [-40, -60, -80, \text{ and } -100]\text{mmHg}$, over interval $t \in (0,1]$

t	FRDTM 40	FVIM 40	FRDTM 60	FVIM 60	FRDTM 80	FVIM 80	FRDTM 100	FVIM 100
0	16.66670000	16.66670000	16.66670000	16.66670000	16.66670000	16.66670000	16.66670000	16.66670000
0.1	22.45411594	22.45411596	27.15815324	27.15815325	31.86219057	31.86219057	36.56622788	36.56622788
0.2	23.29054795	23.29054796	28.67444089	28.67444089	34.05833384	34.05833383	39.44222677	39.44222678
0.3	23.81659557	23.81659556	29.62806233	29.62806232	35.43952907	35.43952908	41.25099585	41.25099585
0.4	24.20582437	24.20582438	30.33365804	30.33365803	36.46149170	36.46149170	42.58932537	42.58932535
0.5	24.51659610	24.51659609	30.89702638	30.89702638	37.27745669	37.27745669	43.65788697	43.65788697
0.6	24.77608652	24.77608652	31.36743178	31.36743178	37.95877704	37.95877705	44.55012230	44.55012230
0.7	24.99927240	24.99927239	31.77202412	31.77202412	38.54477584	38.54477584	45.31752755	45.31752756
0.8	25.19532612	25.19532611	32.12743117	32.12743117	39.05953622	39.05953622	45.99164128	45.99164128
0.9	25.37029032	25.37029032	32.44460705	32.44460705	39.51892380	39.51892380	46.59324055	46.59324054
1.0	25.52836519	25.52836519	32.73116589	32.73116588	39.93396658	39.93396658	47.13676728	47.13676727

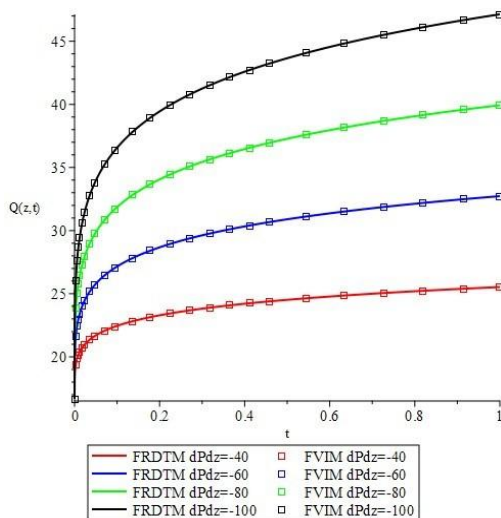


Fig. 6. 2D Plot Showing the Variation of Blood Flow Rate $Q(z,t)$ obtained by FRDTM and FVIM for Fractional-Order $\alpha = 0.25$, with fixed cross-sectional area $S = 1.0\text{cm}^2$ and varying Pressure gradients $\Delta p = [-40, -60, -80, \text{ and } -100]\text{mmHg}$, over the interval $t \in (0,1]$.

Table 6 and 6 presents the computed blood flow rate $Q(z,t)$ obtained using FRDTM and FVIM for the fractional order $\alpha = 0.25$, with a fixed cross-sectional area $S = 1.0\text{cm}^2$ and varying pressure gradients $\Delta p = (-40, -60, -80,$

and $-100)\text{mmHg}$ over the time interval $t \in (0,1]\text{s}$. The table and the figure shows that $Q(z,t)$ is seen to increase gradually with time, but at a slower rate compared with the classical case ($\alpha = 1.0$). The fractional order $\alpha = 0.25$ regime represents blood with strong viscoelastic properties, where inertia and elasticity combine to delay the build-up of flow. This behaviour aligns with the characteristics of non-Newtonian, where the system possesses long-term memory, meaning that current flow depends not only on the instantaneous pressure gradient but also on the history of deformation. At each pressure gradient, the flow rate starts from the same initial value ($16.67\text{cm}^2/\text{s}$) and increases modestly over time (meaning, at $\Delta p = -40\text{mmHg}$, Q rises from $16.67\text{cm}^2/\text{s}$ to $22.45411594\text{cm}^2/\text{s}$ and by $t = 1.0\text{s}$, a 120% increase. Also, at $\Delta p = -100\text{mmHg}$, Q grows from $16.67\text{cm}^2/\text{s}$ to $47.13676727\text{cm}^2/\text{s}$ corresponding to a 338% increase over the same interval. While these increases are significant, they remain lower than those observed at $\alpha = 1.0$, confirming that fractional damping slows the dynamic response. The flow rate remains directly proportional to the pressure gradient, preserving the pressure-flow linearity described by Poiseuille's law, but with temporal retardation due to the fractional order. The close numerical match between FRDTM and FVIM results (identical up to at least six decimal places) validates the consistency of FRDTM methods.

Table 7. Numerical results of Blood Flow Rate $Q(x, t)$ computed by FRDTM and FVIM for Fractional-Orders $\alpha = 0.25, 0.50, 0.75$ and 1.0 , with Cross-sectional Area $S = 1.0\text{cm}^2$ and Pressure Gradient $\Delta p = -60\text{mmHg}$ over the interval $t \in (0, 1]$

t	FRDTM 0.25	FVIM 0.25	FRDTM 0.5	FVIM 0.5	FRDTM 0.75	FVIM 0.75	FRDTM 1.0	FVIM 1.0
0	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000
0.1	27.16073637	27.16073637	23.40404871	23.40404870	20.56616053	20.56616053	18.74792700	18.74792700
0.2	28.67692040	28.67692039	25.76388338	25.76388338	22.99379036	22.99379036	20.73644268	20.73644267
0.3	29.63047665	29.63047665	27.43132219	27.43132218	24.98290601	24.98290601	22.63939431	22.63939430
0.4	30.33602415	30.33602415	28.74924020	28.74924020	26.70320580	26.70320580	24.46046363	24.46046363
0.5	30.89935400	30.89935400	29.84877669	29.84877669	28.23125680	28.23125681	26.20317398	26.20317399
0.6	31.36972727	31.36972726	30.79640287	30.79640288	29.61090295	29.61090295	27.87089708	27.87089708
0.7	31.77429195	31.77429194	31.63116484	31.63116484	30.87059003	30.87059003	29.46685953	29.46685954
0.8	32.12967473	32.12967471	32.37822852	32.37822851	32.03027165	32.03027165	30.99414918	30.99414917
0.9	32.44682893	32.44682893	33.05489048	33.05489048	33.10471474	33.10471471	32.45572091	32.45572091
1.0	32.73336819	32.73336818	33.67361511	33.67361512	34.10528295	34.10528296	33.85440253	33.85440252

The fractional orders considered in this study ($\alpha = 0.25, 0.50$, and 0.75) are selected to represent different levels of viscoelastic behaviour in the system. Lower values of α correspond to stronger memory effects and more pronounced viscoelasticity, while values closer to unity indicate weaker memory and a transition toward classical Newtonian behaviour. This range allows for a systematic investigation of how fractional dynamics influence the temporal evolution of blood flow.

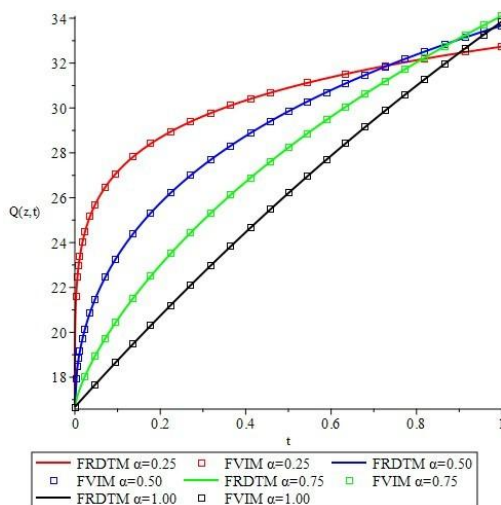


Fig. 7. 2D Plot Showing the Variation of Blood Flow Rate $Q(z, t)$ obtained by FRDTM and FVIM for different

Table 8. Numerical Results of Blood Flow Rate $Q(z, t)$ computed by FRDTM and FVIM for Fractional-Orders $\alpha = 0.25, 0.50, 0.75$ and 1.0 , with $S = 0.5\text{cm}^2$ and $\Delta p = -40\text{mmHg}$ over interval $t \in (0, 1]$

t	FRDTM 0.25	FVIM 0.25	FRDTM 0.5	FVIM 0.5	FRDTM 0.75	FVIM 0.75	FRDTM 1.0	FVIM 1.0
0	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000
0.1	14.56595021	14.56595021	15.20596295	15.20596295	15.77202897	15.77202898	16.17494692	16.17494693
0.2	14.32448311	14.32448311	14.76711104	14.76711103	15.26166677	15.26166676	15.72460276	15.72460276
0.3	14.18274675	14.18274676	14.47758946	14.47758947	14.86959670	14.86959670	15.31218058	15.31218058
0.4	14.08612335	14.08612334	14.26018383	14.26018382	14.54900444	14.54900443	14.93448712	14.93448712
0.5	14.01728915	14.01728914	14.08623517	14.08623517	14.27830478	14.27830479	14.58859797	14.58859798

Fractional-Orders $\alpha = 0.25, 0.50, 0.75$ and 1.0 , with fixed Cross-sectional Area $S = 1.0\text{cm}^2$ and Pressure Gradient $\Delta p = -60\text{mmHg}$, over the interval $t \in (0, 1]$.

Table 7 and Figure 7 presents the computed blood flow rate $Q(z, t)$ using the Fractional Reduced Differential Transform Method (FRDTM) and the Fractional Variational Iteration Method (FVIM) for fractional orders $\alpha = 0.25, 0.50, 0.75$ and 1.0 , at a fixed cross-sectional area $S = 1.0\text{cm}^2$ and pressure gradient $\Delta p = -60\text{mmHg}$ over the time range $t \in (0, 1]\text{s}$. At the initial time ($t = 0\text{s}$), the flow rate starts from the same baseline value $Q = 16.67\text{cm}^2/\text{s}$ for all fractional orders, representing the resting or steady-state condition. As time progresses, the discharge gradually increases across all cases, but the rate of increase depends on the fractional order. For $\alpha = 0.25$, the flow rises from $16.67\text{cm}^2/\text{s}$ to approximately $32.73336819\text{cm}^2/\text{s}$ by $t = 1.0\text{s}$, indicating a more gradual and damped response typical of viscoelastic (sub-diffusive) flow. In contrast, the classical (integer-order) case ($\alpha = 1.0$) reaches $26.14680529\text{cm}^2/\text{s}$, showing a slightly faster buildup in discharge due to the absence of memory effects. Intermediate fractional orders ($\alpha = 0.5$ and 0.75) display transitional behavior, where the response combines both elastic and viscous characteristics, resulting in moderate temporal growth. The differences between FRDTM and FVIM remain negligible (up to four decimal places), confirming that both numerical approaches yield consistent and highly accurate results.

t	FRDTM 0.25	FVIM 0.25	FRDTM 0.5	FVIM 0.5	FRDTM 0.75	FVIM 0.75	FRDTM 1.0	FVIM 1.0
0.6	13.96870830	13.96870831	13.94161585	13.94161586	14.04503059	14.04503059	14.27183501	14.27183502
0.7	13.93657719	13.93657719	13.81826328	13.81826329	13.84109071	13.84109069	13.98174562	13.98174562
0.8	13.91882490	13.91882491	13.71115175	13.71115175	13.66084107	13.66084108	13.71608370	13.71608370
0.9	13.91429159	13.91429159	13.61697171	13.61697169	13.50014637	13.50014638	13.47279229	13.47279229
1.0	13.92233926	13.92233924	13.53347558	13.53347558	13.35586687	13.35586687	13.24998766	13.24998767

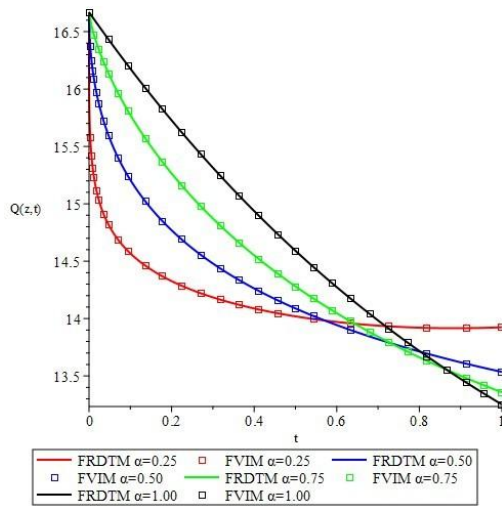


Fig. 8. 2D Plot Showing the Variation of Blood Flow Rate $Q(z, t)$ obtained by FRDTM and FVIM for different Fractional-Orders $\alpha = 0.25, 0.50, 0.75$ and 1.0 , with fixed Cross-sectional Area $S = 0.5\text{cm}^2$ and Pressure Gradient $\Delta p = -40\text{mmHg}$, over $t \in (0, 1]$

Table 8 and Figure 8 present the variation of the blood flow rate $Q(z, t)$ computed using the FRDTM and the FVIM for fractional orders $\alpha = 0.25, 0.50, 0.75$ and 1.0 . The simulation was performed at a fixed cross-sectional area $S = 0.5\text{cm}^2$ and a constant pressure gradient $\Delta p = -40\text{mmHg}$, over $t \in (0, 1]\text{s}$. At $t = 0\text{s}$, the initial flow rate is $Q(0) =$

$16.67\text{cm}^2/\text{s}$ for all fractional orders, representing the initial condition before the flow begins to decay. As time progresses, the flow rate Q decreases steadily across all cases, reflecting a gradual loss of flow intensity due to the applied negative pressure gradient. The rate of this decay depends strongly on the fractional order: it is faster for smaller α values and slows down as α increases. For instance, $t = 1.0\text{s}$, the flow rate drops to approximately $13.92233926\text{cm}^2/\text{s}$, $13.92233924\text{cm}^2/\text{s}$ for $\alpha = 0.25$ and to about $13.24998767\text{cm}^2/\text{s}$ for $\alpha = 1.0\text{s}$. This shows that lower fractional orders correspond to stronger damping or resistive effects, leading to a quicker stabilization of the flow. This pattern is the opposite of what was observed for $S = 1.0\text{cm}^2$, emphasizing how significantly the fractional order influences memory and relaxation effects in the fractional blood flow model. When α is small (e.g., 0.25), the system retains more memory of past states, producing an earlier and sharper reduction in Q . In contrast, as α approaches 1.0 , the behaviour becomes more Newtonian, and the decay in flow rate occurs more smoothly and linearly. Overall, fractional differentiation effectively modulates the transient response of the flow, smaller α values amplify viscous resistance and energy dissipation, while larger α values lessen these effects. Furthermore, the results obtained using FRDTM closely match those from FVIM for all fractional orders, confirming the numerical reliability, stability, and convergence of both methods.

Table 9. Numerical Results of Blood Flow Rate $Q(z, t)$ computed by FRDTM and FVIM for Fractional-Orders $\alpha = 0.25, 0.50, 0.75$ and 1.0 , with $S = 0.5\text{cm}^2$ and $\Delta p = -100\text{mmHg}$, over $t \in (0, 1]$

t	FRDTM 0.25	FVIM 0.25	FRDTM 0.5	FVIM 0.5	FRDTM 0.75	FVIM 0.75	FRDTM 1.0	FVIM 1.0
0	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000	16.67000000
0.1	20.40803624	20.40803623	19.26820377	19.26820376	18.26006686	18.26006687	17.54248902	17.54248902
0.2	20.83807772	20.83807774	20.04977828	20.04977830	19.16899790	19.16899789	18.34453073	18.34453073
0.3	21.09050350	21.09050350	20.56540255	20.56540255	19.86725617	19.86725616	19.07903514	19.07903516
0.4	21.26258525	21.26258524	20.95259174	20.95259174	20.43821587	20.43821587	19.75168939	19.75168939
0.5	21.38517571	21.38517570	21.26238607	21.26238608	20.92031914	20.92031915	20.36770164	20.36770163
0.6	21.47169586	21.47169587	21.51994627	21.51994626	21.33576947	21.33576949	20.93184152	20.93184152
0.7	21.52891986	21.52891988	21.73963136	21.73963134	21.69897684	21.69897683	21.44847703	21.44847704
0.8	21.56053585	21.56053586	21.93039196	21.93039195	22.01999287	22.01999288	21.92160838	21.92160839
0.9	21.56860948	21.56860949	22.09812219	22.09812219	22.30618256	22.30618256	22.35489890	22.35489890
1.0	21.55427700	21.55427699	22.24682483	22.24682484	22.56313756	22.56313755	22.75170341	22.75170342

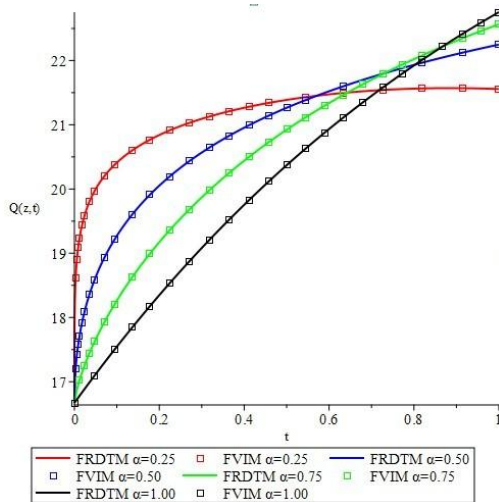


Fig. 9. 2D Plot Showing the Variation of Blood Flow Rate $Q(z, t)$ obtained by FRDTM and FVIM for different Fractional-Orders $\alpha = 0.25, 0.50, 0.75$ and 1.0 , with fixed Cross-sectional Area $S = 0.5\text{cm}^2$ and Pressure Gradient $\Delta p = -100\text{mmHg}$, over $t \in (0, 1]$

Table 9 and Figure 9 presents the numerical results of the blood flow rate $Q(z, t)$ obtained using the FRDTM and FVIM for fractional $\alpha = 0.25, 0.50, 0.75$ and 1.0 , at a fixed cross-sectional area $S = 0.5\text{cm}^2$ and a pressure gradient $\Delta p = -100\text{mmHg}$, over the time range $t \in (0, 1]$. As seen from the table and the plots, at the initial time $t = 0\text{s}$, the flow rate begins at $Q = 16.67\text{cm}^2/\text{s}$ for all fractional orders, representing the initial steady state before the applied pressure gradient induces acceleration in the flow. As time progresses, the flow rate increases consistently for all fractional orders, indicating that a larger negative pressure gradient enhances the driving force and, consequently, the velocity and discharge through the vessel. However, the rate and magnitude of this increase depend strongly on the fractional order α . At smaller fractional orders, such as $\alpha = 0.25$, the flow rate rises more slowly, reaching approximately $21.5543\text{cm}^2/\text{s}$ at $t = 1.0\text{s}$. For larger fractional orders (e.g., $\alpha = 1.0$), the flow rate increases more rapidly and attains a higher peak rate of about $22.7517\text{cm}^2/\text{s}$ at the same time, approaching the behaviour of a purely Newtonian fluid. FRDTM correlates well with the FVIM across all cases confirm the reliability and convergence of FRDTM. The minimal difference between their outputs demonstrates that each method effectively captures the dynamic response of the fractional blood flow system under varying pressure gradients.

5. CONCLUSION

In this study, a fractional-order mathematical model was proposed to describe unsteady blood flow in a compliant artery by incorporating Caputo fractional derivatives, enhancing the representation of viscoelastic and memory effects relative to integer-order models. Analytical solutions were obtained using the Fractional Reduced Differential Transform Method and validated through comparison with the Fractional Variational Iteration Method. The numerical investigation demonstrates that the flow rate increases with time across all considered cases and that the fractional order significantly affects the temporal behaviour of the flow. Lower

fractional orders exhibit more pronounced memory effects, while higher orders yield a faster response and approach classical Newtonian behaviour as the order approaches unity. The consistent agreement between the two solution methods confirms the accuracy and stability of the proposed approach.

Despite these contributions, the present one-dimensional model has certain limitations. It neglects radial velocity components, secondary flow structures, and possible turbulence effects that may arise in complex vascular geometries. While the model provides valuable insight into global hemodynamic behaviour, future studies may extend the analysis to two- or three-dimensional formulations for enhanced physiological accuracy.

These results highlight the importance of fractional modeling in capturing realistic blood flow dynamics and provide a useful mathematical framework for future cardiovascular flow studies.

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